

# Is magnetic field due to an electric current a relativistic effect?

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**Abstract.** Several authors have asserted that the magnetic field due to an electric current is a relativistic effect. This assertion is based on the fact that if one assumes that the interaction between electric charges is entirely due to the electric field, then the relativistic force transformation equations make it imperative that a second field—the magnetic field—is present when the charges are moving. However, as is shown in this paper, if one assumes that the interaction between moving electric charges is entirely due to the magnetic field, then the same relativistic force transformation equations make it imperative that a second field—this time the electric field—is also present. Therefore, since it is impossible to interpret both the electric and the magnetic field as relativistic effects, one must conclude that neither field is a relativistic effect. The true meaning of the calculations demonstrating the alleged relativistic nature of the magnetic field and of the calculations presented in this paper is, therefore, that the idea of a single force field, be it magnetic or electric, is incompatible with the relativity theory.

**Résumé.** Il y a l'opinion que le champ magnétique du courant électrique est un effet relativiste. La base de cette opinion est que si on accepte que l'interaction entre des charges électrique dépend seulement du champ électrique, et si les charges sont en mouvement, les equations relativistes de transformation des forces demandent la présence d'un deuxième champ—du champ magnétique. On démontre ici que si l'on présume que l'interaction entre des charges électriques dépendent seulement d'un champ magnétique, les même equations de transformation des forces relativistes rendent nécessaire la présence d'un deuxième champ, mais cette fois du champ électrique. Cela montre que ni l'un ni l'autre de ces champs est un effet relativiste puisqu'il est impossible d'interpréter les deux champs en même temps comme des effets relativistes. La vraie signification des calculs qui semble indiquer la nature relativiste du champs magnétique, comme des calculs présentés ci-dessous, est que l'existence d'un seul champ, que ce soit électrique ou magnétique, n'est pas compatible avec la théorie de relativité.

## 1. Introduction

In several electricity and magnetism textbooks [1] the authors assert that the magnetic field due to an electric current is a relativistic effect. This assertion is based on the fact that if one assumes that the interaction between electric charges is entirely due to the electric field, then the force transformation equations of the special relativity theory demand the existence of the magnetic field.

It is shown in this paper that one could assert with equal justification that the electric field rather than the magnetic field is a relativistic effect. Therefore, since it is impossible for both fields to be relativistic effects, neither field should be regarded as a relativistic effect.

## 2. Deducing the existence of the electric field on the basis of the relativistic force transformation equations

Consider two very long ('infinitely long') line charges of opposite polarity adjacent to each other along their entire length. Let the magnitude of the line charge density in

each line charge be  $\lambda$ . Let the positive line charge move with velocity  $\mathbf{v} = v\mathbf{i}$  along the  $x$  axis in the positive direction of the axis and let the negative line charge move with velocity  $\mathbf{v} = -v\mathbf{i}$  along the  $x$  axis in the negative direction of the axis. Let us now assume that a positive point charge  $q$  is present in the  $xy$  plane at a distance  $R$  from the line charges (the  $x$  axis) and let us assume that it moves with velocity  $\mathbf{v}$  in the positive direction of the  $x$  axis.

In the laboratory reference frame the two line charges constitute a current,  $2\lambda v$ . By Ampere's law, the magnetic flux density field that this current produces at the location of  $q$  is

$$\mathbf{B} = \mu_0 \frac{\lambda v \times \mathbf{R}}{\pi R^2}, \quad (1)$$

where  $\mathbf{R}$  is directed toward  $q$ . The force exerted by  $\mathbf{B}$  on  $q$  is

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q \left( \mathbf{v} \times \mu_0 \frac{\lambda v \times \mathbf{R}}{\pi R^2} \right), \quad (2)$$

or

$$\mathbf{F} = -\mu_0 \frac{q\lambda v^2}{\pi R^2} \mathbf{R}. \quad (3)$$

Let us now look at the two line charges and the point charge from a reference frame  $\Sigma'$  moving with velocity  $\mathbf{v} = v\mathbf{i}$  relative to the laboratory. The point charge  $q$  is stationary in this reference frame and therefore experiences no magnetic force at all.

However, according to the relativistic force transformation equations [2], if  $q$  experiences a radial force,  $\mathbf{F}$ , in the laboratory reference frame, then it must experience a radial force

$$\mathbf{F}' = \mathbf{F}(1 - v^2/c^2)^{-1/2} \quad (4)$$

in the moving reference frame ( $c$  is the velocity of light). By equation (3), this force is then

$$\mathbf{F}' = -\mu_0 \frac{q\lambda v^2}{\pi R^2(1 - v^2/c^2)^{1/2}} \mathbf{R}. \quad (5)$$

Of course, equation (5) is not really meaningful unless  $\lambda$  in it is converted to  $\lambda'$  pertaining to the moving reference frame  $\Sigma'$ . For making the conversion, we take into account that since in the laboratory reference frame both line charges move, they both are Lorentz contracted<sup>†</sup>, so that the magnitude of the charge density of the positive and negative line charge in the laboratory frame is

$$\lambda = \frac{\lambda_0}{(1 - v^2/c^2)^{1/2}}, \quad (6)$$

where  $\lambda_0$  is the magnitude of the *proper* line charge density of the two line charges (that is, the density measured in a reference frame where the line charge under consideration is stationary).

We also take into account that, since the positive line charge is at rest in  $\Sigma'$ , its density there is

$$\lambda'_+ = \lambda_0. \quad (7)$$

Finally, we take into account that the velocity of the negative line charge in  $\Sigma'$  is, by the velocity addition rule of the relativity theory [4],

$$v'_- = \frac{2v}{1 + v^2/c^2}, \quad (8)$$

so that the line charge density of the negative line charge in  $\Sigma'$  is

$$\begin{aligned} \lambda'_- &= -\frac{\lambda_0}{(1 - v'^2/c^2)^{1/2}} \\ &= -\frac{\lambda_0}{[1 - (4v^2/c^2)/(1 + v^2/c^2)^2]^{1/2}}, \end{aligned} \quad (9)$$

<sup>†</sup> The method for converting  $\lambda$  to  $\lambda'$  that follows is the customary method used in many electricity and magnetism textbooks. However, this method is open to criticisms because it is based on a debatable use of Lorentz length contraction. As we now know, the significance of Lorentz contraction for determining length, shape and volume of moving bodies is far from clear. Some of the works dealing with this subject are given in [3]. An alternative, unquestionably rigorous, conversion of  $\lambda$  to  $\lambda'$  based entirely on Lorentz–Einstein transformation equations of relativistic electrodynamics is presented later on in this paper.

or

$$\lambda'_- = -\frac{\lambda_0(1 + v^2/c^2)}{(1 - v^2/c^2)}. \quad (10)$$

The total line charge density in  $\Sigma'$  is therefore

$$\lambda' = \lambda'_+ + \lambda'_- = \lambda_0 - \lambda_0 \frac{(1 + v^2/c^2)}{(1 - v^2/c^2)}, \quad (11)$$

or

$$\lambda' = -\frac{2\lambda_0 v^2}{c^2(1 - v^2/c^2)}, \quad (12)$$

which, with equation (6), gives

$$\lambda' = -\frac{2\lambda v^2}{c^2(1 - v^2/c^2)^{1/2}}. \quad (13)$$

Substituting equation (13) into equation (5), we obtain for the force on the point charge  $q$  in the moving reference frame  $\Sigma'$

$$\mathbf{F}' = \mu_0 \frac{c^2 q \lambda'}{2\pi R^2} \mathbf{R}, \quad (14)$$

and, since  $\mu_0 c^2 = 1/\epsilon_0$ ,

$$\mathbf{F}' = \frac{q \lambda'}{2\pi \epsilon_0 R^2} \mathbf{R}, \quad (15)$$

which is exactly what we would have obtained for the force exerted on  $q$  in  $\Sigma'$  by the electric field due to the line charge of density  $\lambda'$  (note that  $\lambda'$  is *negative*, so that the field is directed toward the two line charges).

## 2.1. Finding $\lambda'$ from Lorentz–Einstein charge density transformation equation

As has been pointed out above (see footnote below left), the method of converting  $\lambda$  into  $\lambda'$  by means of Lorentz contraction is open to criticisms, to say nothing of its complexity. A preferable method for converting  $\lambda$  into  $\lambda'$  is to use the Lorentz–Einstein transformation equation for charge density [5]

$$\rho' = \gamma(\rho - vJ_x/c^2), \quad (16)$$

where  $\gamma = 1/(1 - v^2/c^2)^{1/2}$  and  $J_x$  is the  $x$  component of the current density. The charge density  $\rho$  in the laboratory reference frame is  $\rho = (\lambda_+ + \lambda_-)/S = 0$ , and the current density is  $J_x = 2\lambda v/S$ , where  $S$  is the cross-sectional area of the positive and the negative line charge. Substituting  $\rho$  and  $J_x$  into equation (16) and multiplying by  $S$ , we immediately obtain

$$\lambda' = -\gamma \frac{2\lambda v^2}{c^2} = -\frac{2\lambda v^2}{c^2(1 - v^2/c^2)^{1/2}}, \quad (17)$$

which is the same as equation (12) obtained earlier with considerably greater effort by using Lorentz contraction and the velocity addition rule.

## 2.2. An alternative method for obtaining $E'$

It is instructive to derive the electric field responsible for the force in equation (15) without using the force transformation. We start with the Lorentz–Einstein transformation equations for the electric field [6]

$$E'_x = E_x, \quad (18)$$

$$E'_y = \gamma(E_y - vB_z), \quad (19)$$

$$E'_z = \gamma(E_z - vB_y). \quad (20)$$

According to equation (1), in the laboratory reference frame  $B_y = 0$ , and

$$B_z = \mu_0 \frac{\lambda v}{\pi R}. \quad (21)$$

The electric field components in the laboratory reference frame are  $E_x = E_y = E_z = 0$ , because the total charge density  $\lambda_+ + \lambda_- = 0$ . By equations (18)–(20), the electric field in  $\Sigma'$  is therefore

$$E'_y = -\gamma \mu_0 \frac{\lambda v^2}{\pi R} = -\mu_0 \frac{\lambda v^2}{\pi R(1 - v^2/c^2)^{1/2}}. \quad (22)$$

Using now equation (17) to replace  $\lambda$  by  $\lambda'$  and remembering that  $\mu_0 c^2 = 1/\epsilon_0$ , we promptly obtain

$$E'_y = \frac{\lambda'}{2\pi\epsilon_0 R}, \quad (23)$$

which is the same as the electric field indicated by equation (15).

## 3. Discussion

As is clear from equations (1)–(15) and (23), relativistic force transformation equations demand the presence of an electric field when the interactions between electric charges are assumed to be entirely due to a magnetic force. We could interpret this result as the evidence that the electric field is a relativistic effect. But the well known fact that similar calculations demand the presence of a magnetic field, if the interactions between the charges are assumed to be entirely due to an electric force, makes such an interpretation impossible (unless we are willing to classify both the magnetic and the electric field as relativistic effects, which is absurd). We must conclude therefore that neither the magnetic nor the electric field is a relativistic effect†.

The only correct interpretation of our results must then be that interactions between electric charges that are either entirely velocity independent or entirely velocity dependent is incompatible with the relativity theory. Both fields—the electric field (producing a force independent of the velocity of the charge experiencing

the force) and the magnetic field (producing a force dependent on the velocity of the charge experiencing the force)—are necessary to make interactions between electric charges relativistically correct. By inference then, any force field compatible with the relativity theory must have an electric-like ‘subfield’ and a magnetic-like ‘subfield’.

## References

- [1] Since this article is not meant to be a criticism of any textbook, I believe that no specific references to such textbooks are needed.
- [2] See, for example, Rosser W G V 1968 *Classical Electromagnetism via Relativity* (New York: Plenum) pp 14–15.
- [3] Terrell J 1959 Invisibility of the Lorentz Contraction *Phys. Rev.* 116 1041–5  
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—— Retardation and relativity: The case of a moving line charge *Am. J. Phys.* 63 454–9.
- [4] See, for example, reference [2], pp 9–10.
- [5] See, for example, reference [2], p 168.
- [6] See, for example, reference [2], p 157.
- [7] Jackson J D 1975 *Classical Electrodynamics* 2nd ed (New York: Wiley) pp 578–81

† In this connection it should be mentioned that J D Jackson, in [7], points out on the basis of a general analysis of relativistic relations that it is impossible to derive magnetic field from Coulomb’s law of electrostatics combined with equations of the special relativity theory without some additional assumptions.