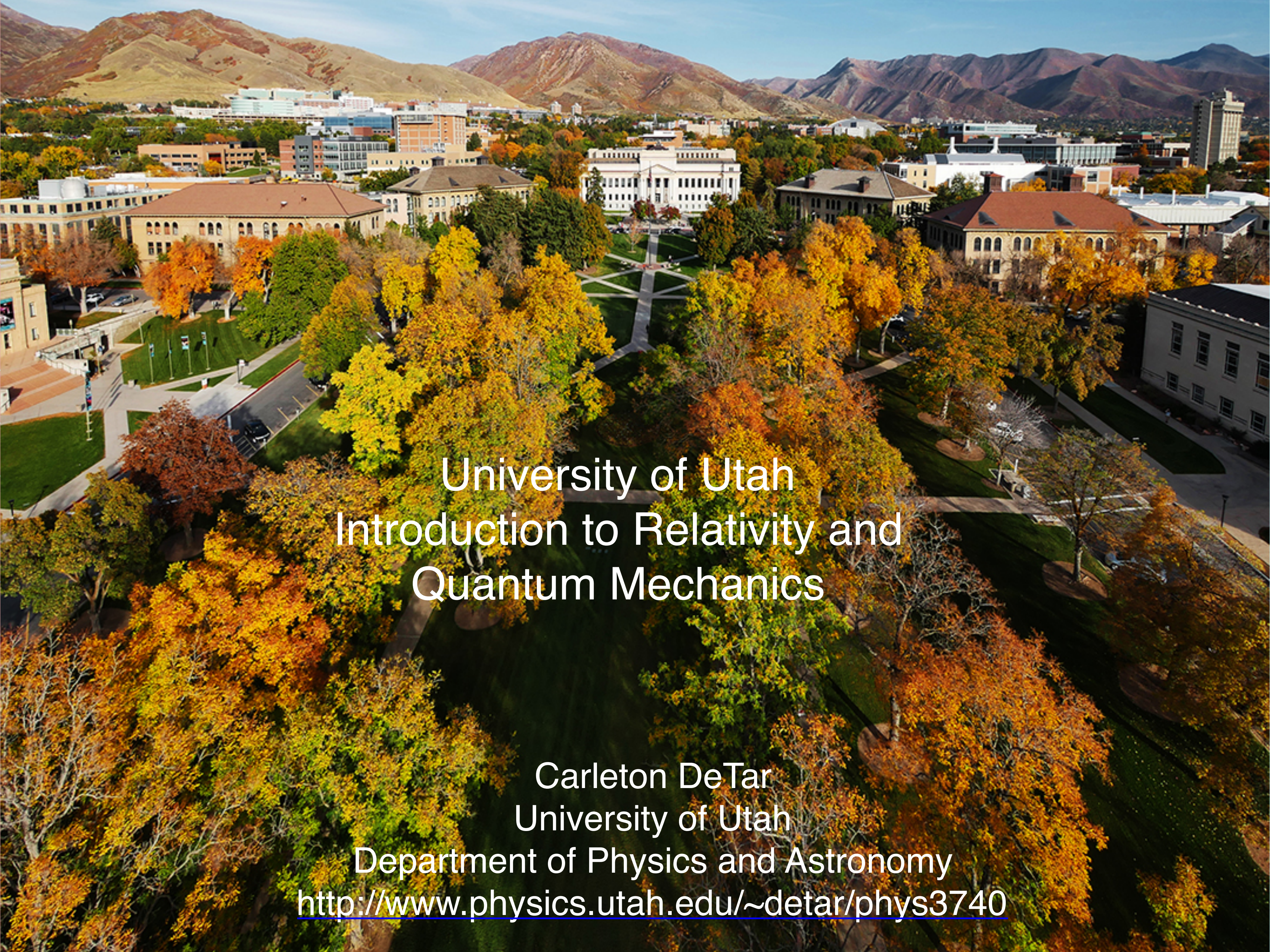

University of Utah Introduction to Relativity and Quantum Mechanics

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Lorentz transf

$$r' = \underline{\Lambda}\, r$$

1 dim

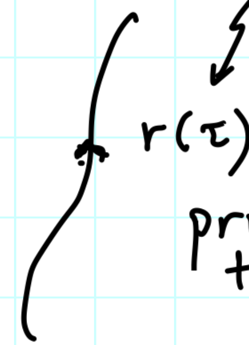
$$\begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}$$

$$S \rightarrow r = (\underline{ct}, \overbrace{x, y, z}^{\vec{r}})$$

Four-vector

$$S' \quad r' = (ct', x', y', z')$$

τ is time on traveler's clock



proper time

Four-velocity

$$u = \frac{dr}{d\tau}$$

$$u = \left(\frac{c\,dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$$

$$\frac{dt}{d\tau} = \gamma \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = \gamma \cdot v_x$$

$$u = (c\gamma, \gamma \cdot \vec{v})$$

four-momentum

$$p = mu = (mc\gamma, \gamma m \vec{v})$$

four-vectors

$$m\frac{dr'}{d\tau} = m\Lambda \frac{dr}{d\tau} \qquad p' = \Lambda p$$

Momentum conservation

$$\Lambda \sum_{\text{initial}} p_i = \Lambda \sum_{\text{final}} p_i$$

$$\sum_{\text{initial}} p_i' = \sum_{\text{final}} p_i'$$

$$p = (\underline{\gamma mc}, \gamma m \vec{v})$$

$$E = \gamma mc^2$$

$$= \underline{\underline{mc^2}} + \frac{1}{2} m v^2$$

$$p = (E/c, \vec{p}) = (\gamma mc, \gamma m \vec{v})$$

$$E_{kin} = E - mc^2$$

momentum $\left[\frac{ML}{T}\right]$

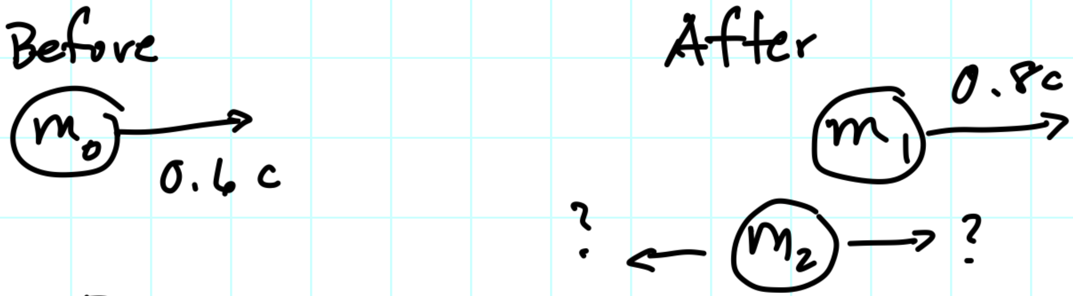
energy $\left[ML^2/T^2\right]$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \simeq \underline{1 + \frac{1}{2} v^2/c^2 + \cdots}$$

$$\sum_{\text{initial}} E_i = \sum_{\text{final}} E_i$$

Harris 2.88

A particle of mass m_0 moves through the lab at $0.6c$. Suddenly, it explodes into two fragments. Fragment 1, mass $0.66m_0$ moves at $0.8c$ in the same direction as the original particle had been moving. Determine the velocity and mass of fragment 2.



$m_1 = 0.66m_0$

$$E_0 = E_1 + E_2$$

$$p_0 = p_1 + p_2$$

$$E_0 = \gamma_0 m_0 c^2$$

$$E_1 = \gamma_1 m_1 c^2$$

$$E_2 = E_0 - E_1 = \gamma_2 m_2 c^2$$

$$\gamma_0 = \frac{1}{\sqrt{1-0.6^2}} = \frac{5}{4}$$

$$p_0 = p_1 + p_2$$

$$p_0 = \gamma_0 m_0 v_0$$

$$p_1 = \gamma_1 m_1 v_1$$

$$p_2 = p_0 - p_1 = \gamma_2 m_2 v_2$$

Lorentz Invariants

$$\Delta s^2 = (\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$
$$= (\Delta ct')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$$

$$p^2 = (E/c)^2 - (\vec{p})^2 = (E'/c)^2 - (\vec{p}')^2$$
$$= m^2c^2$$

$$a \cdot b = a_t b_t - a_x b_x - a_y b_y - a_z b_z$$

invariant!