PHYS 3740 Final Exam

Name: _____ Date: _____

This test is open book and notes, but it is not collaborative. You may not seek or receive assistance from anyone other than course "management".

Please answer in the spaces provided. Answers here must be clearly presented with intermediate steps in a logical order and with notation clearly defined. Use scratch paper if need be to collect your thoughts before writing below.

Some constants

$$\begin{aligned} h &= 6.626 \times 10^{-34}\,\text{J-s} \\ \hbar &= 1.055 \times 10^{-34}\,\text{J-s} \\ c &= 2.998 \times 10^{8}\,\text{m/s} \\ 1\,\text{eV} &= 1.602 \times 10^{-19}\,\text{J} \end{aligned} \qquad \begin{aligned} m_p &= 1.673 \times 10^{-27}\text{kg} \\ m_e &= 9.109 \times 10^{-31}\text{kg} \\ e &= 1.602 \times 10^{-19}\text{C} \end{aligned}$$

1. (20 pts) A neutral hydrogen atom in its ground state is placed in a magnetic field of strength 0.5 T in the positive z direction. This splits the energies of the electron's spin-up and spin-down states. The spin quantum number m_s is defined in the usual way with respect to the same z axis.

   (a) (5 pts) What is the value of m_s for the lower energy state?

   (b) (10 pts) Microwaves of an appropriate frequency cause the electron to flip its spin. What is that frequency in Hz? What is the wavelength in cm?

(c) (5 pts) The proton also carries spin 1/2, but it has a gyromagnetic ratio (g factor) of 5.6. In the same magnetic field, what frequency (Hz) of radiation would cause it to flip its spin?

2. (30 pts) A neutral boron atom has five electrons.

(a) (10 pts) What are the possible quantum numbers n, ℓ, m_ℓ for the outermost electron in the lowest energy state?

(b) (20 pts) Write the three orbital wave functions for each of the sets of possible quantum numbers from part (a). Show the explicit dependence on the spherical polar coordinates r, θ, and ϕ. Assume that the inner electrons screen the charge of the nucleus as much as they can. Ignore interactions among the electrons.

3. (20 pts) Two identical spinless bosons of mass m move in a one-dimensional infinite well of width L for $0 < x < L$. They are both in states of well-defined energy.

   (a) (10 pts) What are the two lowest total energies of the two-particle system? For each of them, give the quantum numbers assigned to the particles.

   (b) (10 pts) For the higher energy in (a), write a properly normalized wave function for two bosons in terms of their coordinates x_1 and x_2.

4. (30 pts) An accelerator produces a beam of pi-meson particles with energy 400 MeV. Take the $+x$ axis to be the direction of the beam.

   (a) (6 pts) The pi meson has a rest mass energy of 140 MeV/c^2. What is the momentum of one the pi mesons in units of MeV/c?

   (b) (6 pts) One of the pi mesons decays to produce two particles. One of them is massless and moves along the y axis with momentum 11 MeV/c. What is the energy in MeV of that particle?

(c) (6 pts) What is the four-momentum in MeV/c of the second particle?

(d) (6 pts) What is the mass in MeV/c^2 of the second particle?

(e) (6 pts) What is the speed of the second particle in units of c?