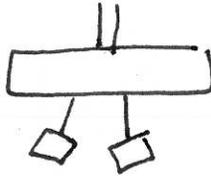


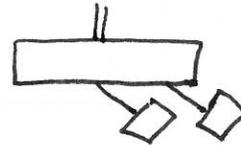
CH 9: Non-Inertial Systems

⇒ ACCELERATING REFERENCE FRAMES RESULT IN FICTITIOUS FORCES FOR OBSERVERS IN THESE FRAMES.

e.g. "fuzzy Dice"



Driving Straight



Left Turn

Dice appear to "FLEE" THE CENTER
"CENTRIFUGAL FORCE"

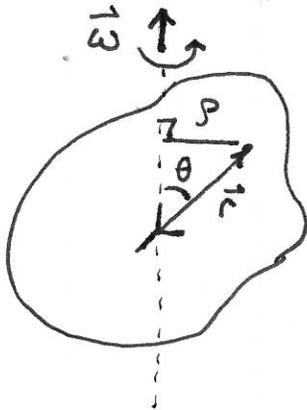
(SAME ROOT AS "Tempus Fugit")

⇒ CAN WE FORMALLY DESCRIBE THESE FICTITIOUS FORCES, & DO PHYSICS IN FRAME OF ACCELERATED OBSERVER?

⇒ A couple of preliminaries

Theorem I) ARBITRARY MOTION OF A RIGID BODY CONTAINING A POINT P CAN BE WRITTEN AS THE SUM OF THE TRANSLATIONAL MOTION OF P PLUS A ROTATION $\vec{\omega}(t)$ ABOUT P.

Theorem II) GIVEN AN OBJECT ROTATING w/ ANGULAR VELOCITY $\vec{\omega}$, THE VELOCITY OF A POINT AT \vec{r} IS GIVEN BY ...

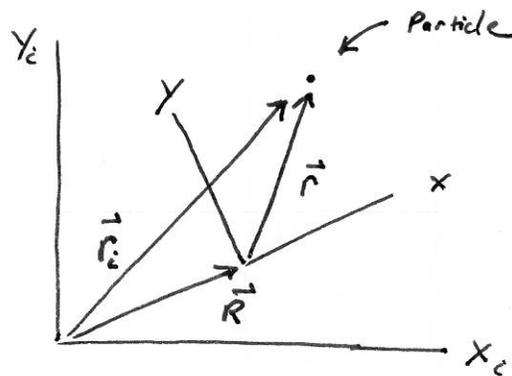


$$v = \omega r$$

$$= \omega r \sin \theta$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

- ⇒ Consider an inertial system w/ basis vectors $\hat{x}_i, \hat{y}_i, \hat{z}_i$
 & system w/ arbitrary acceleration & basis vectors $\hat{x}, \hat{y}, \hat{z}$



$$\vec{r}_i = \vec{R} + \vec{r} \quad (i)$$

- ⇒ We want to take TWO DERIVATIVES w.r.t. time of \vec{r}_i ,
 So we can modify Newton's 2nd.

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

$$\frac{d\vec{r}}{dt} = \left(\frac{dr_x}{dt} \hat{x} + \frac{dr_y}{dt} \hat{y} + \frac{dr_z}{dt} \hat{z} \right) + \left(r_x \frac{d\hat{x}}{dt} + r_y \frac{d\hat{y}}{dt} + r_z \frac{d\hat{z}}{dt} \right) \quad \text{Product Rule}$$

2nd Term: Tan I) With Pas origin, change in unit vectors consists of translation $\vec{R}(t)$ & rotation $\vec{\omega}(t)$ about the origin.

$$\text{Tan II)} \quad \frac{d\vec{A}}{dt} = \vec{\omega} \times \vec{A}$$

Where \vec{A} is any vector of fixed length rotating w/ angular velocity $\vec{\omega}$.

So ...

$$r_x \frac{dx^1}{dt} = r_x (\vec{\omega} \times \hat{x}) = \vec{\omega} \times (r_x \hat{x})$$

$$r_y \frac{dy^1}{dt} = \vec{\omega} \times (r_y \hat{y})$$

$$r_z \frac{dz^1}{dt} = \vec{\omega} \times (r_z \hat{z})$$

$$\therefore \frac{d\vec{r}}{dt} = \left(\frac{dr_x}{dt} \hat{x} + \frac{dr_y}{dt} \hat{y} + \frac{dr_z}{dt} \hat{z} \right) + \vec{\omega} \times \vec{r}$$

⇒ Now, TAKE 2ND TIME DERIVATIVE

$$\frac{d^2\vec{r}}{dt^2} = \underbrace{\left(\frac{d^2r_x}{dt^2} \hat{x} + \dots \right)}_{\vec{a}} + \underbrace{\left(\frac{dr_x}{dt} \frac{d\hat{x}}{dt} + \dots \right)}_{\vec{\omega} \times \vec{v}} + \underbrace{\frac{d\vec{\omega}}{dt} \times \vec{r}}_{\frac{d\vec{\omega}}{dt} \times \vec{r}} + \underbrace{\vec{\omega} \times \frac{d\vec{r}}{dt}}_{\vec{\omega} \times \vec{v} + \vec{\omega} \times (\vec{\omega} \times \vec{r})}$$

$$\frac{d^2\vec{r}}{dt^2} = \vec{a} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \vec{v} + \frac{d\vec{\omega}}{dt} \times \vec{r} \quad (ii)$$

Remember: $\vec{r}, \vec{v}, \vec{a}$ are position, velocity & acceleration w.r.t. accelerating frame.

⇒ BACK TO EQU (i) TAKE TWO DERIVATIVES & multiply by m

$$m \frac{d^2\vec{r}}{dt^2} = m \frac{d^2\vec{R}}{dt^2} + m \frac{d^2\vec{r}}{dt^2}$$

WHAT WE'VE CALLED
" \vec{F} " ALL SEMESTER



$$m \vec{a} = \vec{F} - m \frac{d^2 \vec{R}}{dt^2} - m \vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m \vec{\omega} \times \vec{v} - m \frac{d\vec{\omega}}{dt}$$

$$m \vec{a} = \vec{F} + \underbrace{\vec{F}_{\text{TRANSLATION}} + \vec{F}_{\text{CENTRIFUGAL}} + \vec{F}_{\text{CORIOLIS}} + \vec{F}_{\text{AZIMUTHAL}}}_{\text{"FICTITIOUS FORCES"}}$$

↑
accelerating
frame

↑
real
force

"FICTITIOUS FORCES"

↳ we have a new version of Newton's 2nd,
VALID IN ACCELERATING FRAMES!

$$\vec{F}_{\text{TRANS}} = -m \frac{d^2 \vec{R}}{dt^2}$$

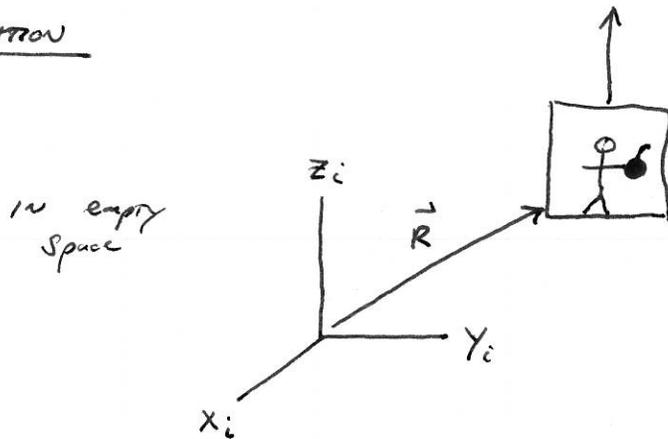
$$\vec{F}_{\text{CENT}} = -m \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{F}_{\text{COR}} = -2m \vec{\omega} \times \vec{v}$$

$$\vec{F}_{\text{AZ}} = -m \frac{d\vec{\omega}}{dt} \times \vec{r}$$

CONSIDER FICTITIOUS FORCES IN TURN

$\vec{F}_{\text{TRANSLATION}}$



$$\frac{d^2 \vec{R}}{dt^2} = 9.8 \text{ m/s}^2 \hat{z}$$
$$= +g \hat{z}$$

$$\vec{F}_{\text{TRANS}} = -m \frac{d^2 \vec{R}}{dt^2} = -mg \hat{z}$$

\Rightarrow ISAC & APPLE experience FICTITIOUS FORCE
WHICH (TO THEM) IS INDISTINGUISHABLE FROM GRAVITY.

\Rightarrow MOTIVATED EINSTEIN "EQUIVALENCE PRINCIPLE"

INERTIAL MASS \iff GRAVITATIONAL MASS

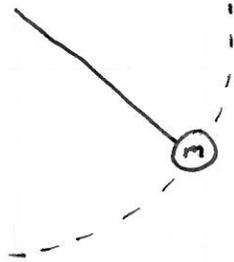
BASIS OF GR!

\Rightarrow WHAT IF ISAC IS HOLDING A FLASHLIGHT?

\Rightarrow WHAT IF YOU WERE BORN IN OUTER SPACE & ONLY EXPERIENCED NORMAL FORCES WHEN ACCELERATING?

$$\underline{\vec{F}_{\text{centrifugal}}} = -m \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Recall Ball on String

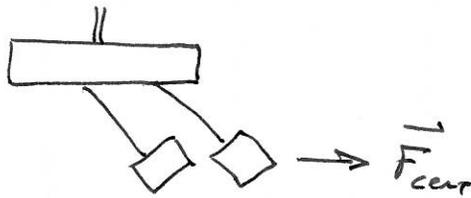


$$m a_{\text{centripetal}} = \frac{m v^2}{r} = m \omega^2 r$$

Directed TOWARDS center of curvature

→ centrifugal "force" is of equal magnitude,
Directed away from center of curvature.

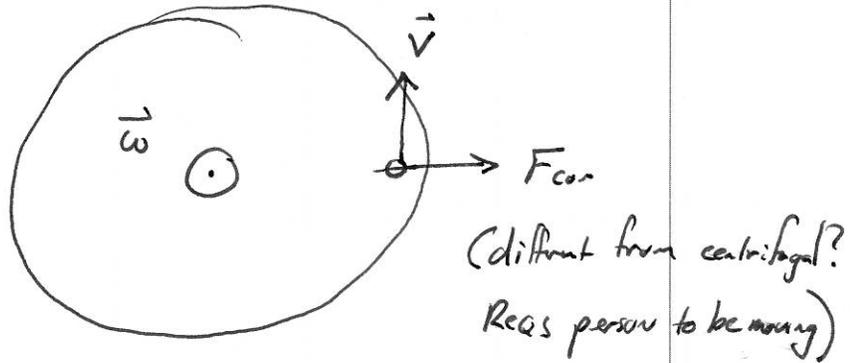
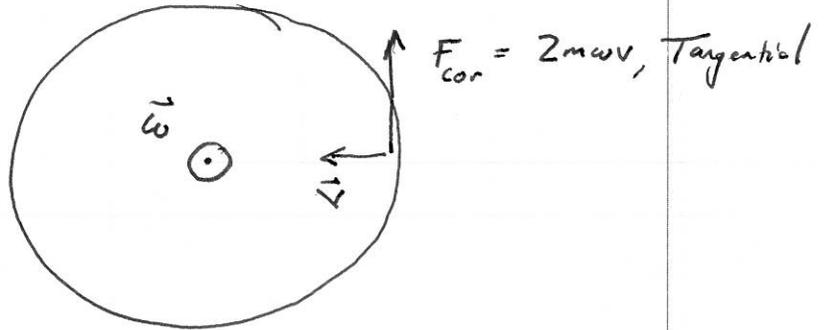
Fuzzy Dice



$-\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is away from center of curvature.

$$\vec{F}_{\text{Coriolis}} = -2m\vec{\omega} \times \vec{v}$$

Carousel, from above



$$\vec{F}_{\text{Azimuthal}} = -m \frac{d\vec{\omega}}{dt} \times \vec{r}$$

If $\vec{\omega}$ increases

$\frac{d\vec{\omega}}{dt}$ is out-of-page

(ie more ccw)

