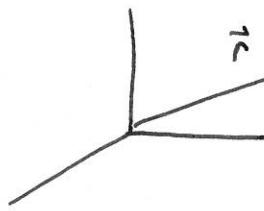


KINEMATICS : Description of Motion

⇒ Already have seen "position vector"

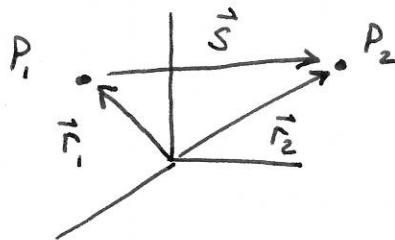


$$\vec{r} \rightarrow P(x, y, z)$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(Not a true vector ---
Doesn't transform correctly
between coord systems)

⇒ What if we displace an object from position \vec{r}_1 to \vec{r}_2



$$\vec{s} = \vec{r}_2 - \vec{r}_1 \quad \text{"Displacement Vector"}$$

↳ True vector

$$\text{Magnitude } |\vec{s}| = \sqrt{\vec{s} \cdot \vec{s}} = \sqrt{s_x^2 + s_y^2 + s_z^2}$$

is distance between P_1 & P_2

⇒ Velocity is rate of change of position/displacement vector

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

Notation $\frac{dx}{dt} = \dot{x} = v_x$, etc

$$V = \sqrt{\vec{v} \cdot \vec{v}} \equiv \text{"speed"}$$

⇒ Acceleration is rate of change of velocity vector

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \quad \frac{d^2x}{dt^2} = \ddot{x} = a_x, \text{ etc}$$

$$a = \sqrt{\vec{a} \cdot \vec{a}} = \text{"acceleration"} \quad \ddot{x}$$

⇒ Constant acceleration

e.g. gravity near Earth's surface

(Projectile Motion)

$$|\vec{a}| = g = 9.81 \frac{m}{s^2}$$

"Acceleration Due to Gravity"

Direction = Down!

Easily integrated $\frac{dv}{dt} = a = \text{constant}$

$$\int_{v_0}^v dv = \int_0^t a dt$$

$$v - v_0 = at$$

$$v = v_0 + at$$

Continuing

$$\frac{dx}{dt} = v_0 + at$$

$$\int_{x_0}^x dx = \int_0^t v_0 dt + \int_0^t at dt$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Also, can solve for t

$$t = \frac{v - v_0}{a} \quad x = x_0 + \frac{v_0(v - v_0)}{a} + \frac{1}{2}a \frac{(v - v_0)^2}{a^2}$$

↓ arithmetic

$$\frac{v^2 - v_0^2}{x - x_0} = 2a$$

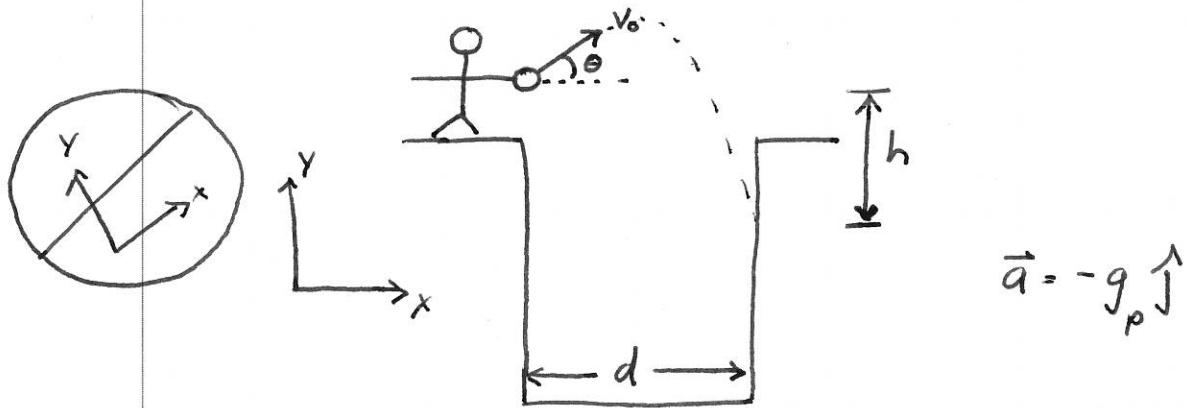
Generalize to 3D

$$\vec{V}(t) = \vec{V}_0 + \vec{a} t$$

$$\vec{r}(t) = \vec{r}_0 + \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

Ex AN ASTRONAUT STANDS BY A CANYON ON ANOTHER PLANET. SHE THROWS A BALL W/ INITIAL SPEED $V_0 = 5.0 \text{ m/s}$, AT AN ANGLE $\theta = 30^\circ$ ABOVE HORIZONTAL. THE BALL STRIKES THE FAR SIDE CANYON WALL, A DISTANCE d HORIZONTALLY AND FALLING A DISTANCE h . NEGLECT AIR RESISTANCE.

- IF THE BALL TAKES 3.0 SEC TO STRIKE THE FAR WALL, WHAT IS d ?
- IF $h = 12.5$ meters, what is g_{planet} ?



→ Apply kinematic relations by component

$$V_x(t) = V_{0x} + \cancel{a_x t}^{\rightarrow 0}$$

$$X(t) = X_0 + V_{0x}t + \cancel{\frac{1}{2} a_x t^2}^{\rightarrow 0} \quad V_{0x} = V_0 \cos \theta$$

$$V_y(t) = V_{0y} + a_y t$$

$$Y(t) = Y_0 + V_{0y}t + \frac{1}{2} a_y t^2 \quad V_{0y} = V_0 \sin \theta$$

$$\downarrow \quad V_x(t) = V_0 \cos \theta \quad X(t) = X_0 + V_0 \cos \theta t$$

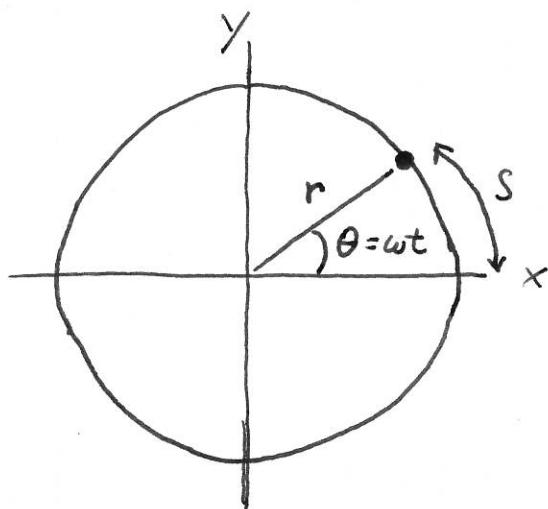
$$V_y(t) = V_0 \sin \theta - g_p t \quad Y(t) = Y_0 + V_0 \sin \theta t - \frac{1}{2} g_p t^2$$

$$a) \quad d = X - X_0 = V_0 \cos \theta t = (5.0 \text{ m/s}) \cos 30^\circ (3 \text{ sec}) = \boxed{12.99 \text{ m}}$$

$$b) \quad -h = Y - Y_0 = V_0 \sin \theta t - \frac{1}{2} g_p t^2$$

$$g_p = \frac{2}{t^2} (h + V_0 \sin \theta t) = \frac{2}{(3.0 \text{ s})^2} (12.5 \text{ m} + (5.0 \text{ m/s}) \sin 30^\circ (3 \text{ sec})) \\ = \boxed{4.44 \text{ m/s}^2}$$

Ex: Uniform Circular Motion (constant radius, constant tangential speed)



"radian" \rightarrow 2π -radians = 360°
 arc length $s = r\theta$ if θ in radians
 ω = angular speed
 greek "little omega"
 units radians/sec

$$V_T = r\omega$$

Tangential Speed

$$x = r \cos \theta = r \cos \omega t$$

$$r^2 = x^2 + y^2$$

$$y = r \sin \theta = r \sin \omega t$$

$$\tan \theta = y/x$$

$$\vec{r} = r \cos \omega t \hat{i} + r \sin \omega t \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -r\omega \sin \omega t \hat{i} + r\omega \cos \omega t \hat{j}$$

NOTE: $\vec{v} \cdot \vec{r} = -r^2 \omega \cos \omega t \sin \omega t + r^2 \omega \sin \omega t \cos \omega t = 0 !$

$\therefore \vec{v} \perp \vec{r}$ velocity tangent to trajectory

$$\vec{a} = \frac{d\vec{v}}{dt} = -r\omega^2 \cos \omega t \hat{i} - r\omega^2 \sin \omega t \hat{j} = -\omega^2 \vec{r}$$

\vec{a} is towards center of circle \rightarrow "centripetal" acceleration

$$|\vec{a}| = \omega^2 r = \frac{V^2}{r}$$

Sketch:

