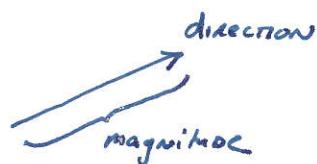


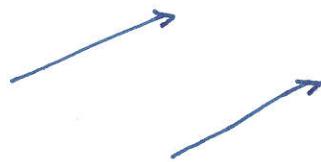
# VECTORS: (Please read Ch 1 of Kleppen)

- Scalars - MAGNITUDE in some units
- Vector - MAGNITUDE & DIRECTION
- [ - Tensor - generalize scalar, vector... much later! ]

⇒ Can represent a vector by an arrow

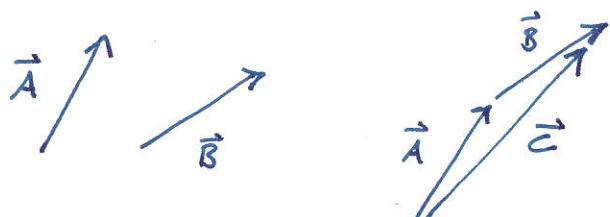


Q: Are THESE vectors DIFFERENT?



⇒ Vector OPERATIONS

ADDITION:  $\vec{A} + \vec{B} = \vec{C}$  (Arrow over letter indicates vector)



"TIP TO TAIL" METHOD

Scalar multiplication:

$$\vec{B} = s \vec{A}$$

$$\vec{A} \quad \vec{B} = 2\vec{A}$$

$$\leftarrow \vec{C} = -1\vec{A} = -\vec{A}$$

If scalar is negative,  
direction flips

Subtraction:  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{C}$



⇒ More useful for quantitative purposes:

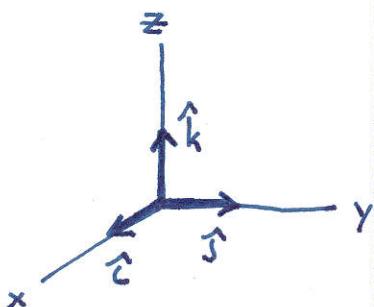
Specify vectors in a particular Coordinate System

e.g. "Cartesian"

- perpendicular ( $\perp$ )  $x, y, z$  axes

- "unit vectors" on each axis

$\hat{i} \hat{j} \hat{k}$  or  $\hat{x} \hat{y} \hat{z}$



Then  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$   
 $= (A_x, A_y, A_z)$



$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$s\vec{A} = sA_x\hat{i} + sA_y\hat{j} + sA_z\hat{k}$$

⇒ Vector Multiplication :      Vector × Vector

2 common products (Are you need !)

- Scalar or dot product

- Vector or cross product

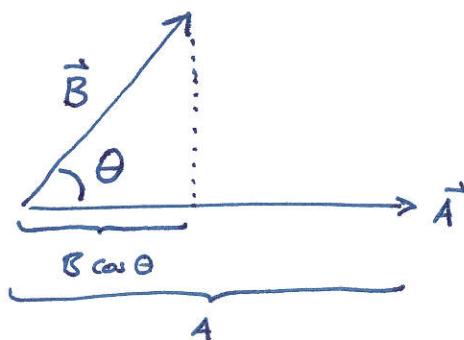
↑  
result

↑  
symbol

Angle between (tail-to-tail)

Scalar Product

$$\vec{A} \cdot \vec{B} = \underbrace{|\vec{A}| |\vec{B}|}_{\text{"magnitude of A"}} \cos \theta = AB \cos \theta$$

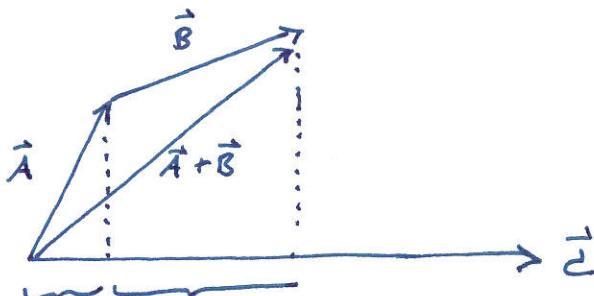


$$\vec{A} \cdot \vec{B} = A (B \cos \theta)$$

= (magnitude of A) × (projection of B on A)

Q: Is scalar product distributive?

$$(\vec{A} + \vec{B}) \cdot \vec{C} \stackrel{?}{=} (\vec{A} \cdot \vec{C}) + (\vec{B} \cdot \vec{C})$$



$$\frac{\vec{A} \cdot \vec{C}}{c} \quad \frac{\vec{B} \cdot \vec{C}}{c}$$

$$\underbrace{\frac{(\vec{A} + \vec{B}) \cdot \vec{C}}{c}}$$

$$\frac{\vec{A} \cdot \vec{C}}{c} + \frac{\vec{B} \cdot \vec{C}}{c} = \frac{(\vec{A} + \vec{B}) \cdot \vec{C}}{c}$$

So, in a Cartesian system

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

Non zero terms in general, but note  $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

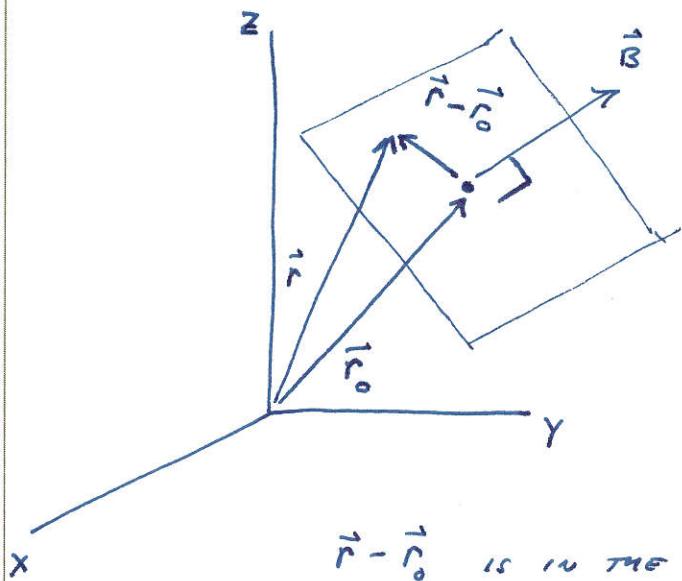
$$\text{So ... } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\text{Also } \vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = (\text{length of } A)^2 \quad \text{Pythagoras!}$$

$$|\vec{A}| = A = \sqrt{\vec{A} \cdot \vec{A}}$$

↳ How we find magnitude of a vector whose components we know.

Ex: Find the equation for a plane containing  $P = (x_0, y_0, z_0)$  & perpendicular to the vector  $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$



$$\begin{aligned}\vec{r}_0 &= x_0 \hat{i} + y_0 \hat{j} + z_0 \hat{k} \\ &= \text{"position vector" of } P \\ &\quad \text{Displacement from origin}\end{aligned}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

$\vec{r} - \vec{r}_0$  is in the plane

$$(\vec{r} - \vec{r}_0) \perp \vec{B}$$

$$(\vec{r} - \vec{r}_0) \cdot \vec{B} = 0$$

$$\vec{r} \cdot \vec{B} = \vec{r}_0 \cdot \vec{B}$$

$$B_x x + B_y y + B_z z = B_x x_0 + B_y y_0 + B_z z_0 = C \quad (\text{constant!})$$

$$B_x x + B_y y + B_z z = C$$

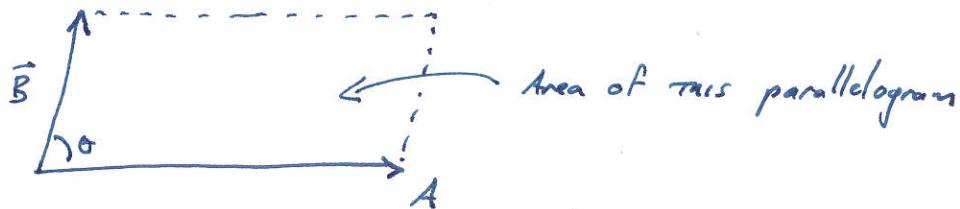
↳ equation of a plane!

↗ A vector specifies a plane  
A plane specifies a vector

$\Rightarrow$  Vector  $\times$  Vector = "vector" product

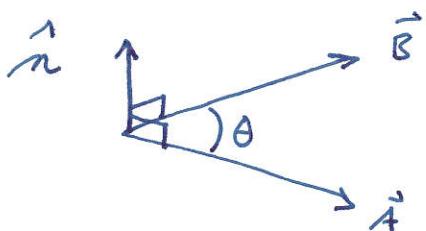
$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$



The Direction of  $\vec{A} \times \vec{B}$  comes from "right-hand rule"

< Demo >



$\hat{n}$  is unit vector  $\perp$  to both  $\vec{A}$  and  $\vec{B}$

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\vec{B} \times \vec{A} = AB \sin \theta (-\hat{n})$$

$$= -\vec{A} \times \vec{B} \quad \text{Non-commutative!}$$

Q: Can a unit vector be  $\perp$  to three vectors,  
 $\vec{A}, \vec{B}$  &  $\vec{C}$ ?

$\Rightarrow$  Vector product in Cartesian Coordinates

1st Note:  $\hat{i} \times \hat{j} = \hat{k}$ ,  $\hat{j} \times \hat{k} = \hat{i}$ ,  $\hat{k} \times \hat{i} = \hat{j}$

So

$$\begin{aligned}\vec{A} \times \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= (A_y B_z - B_y A_z) \hat{i} + (A_z B_x - B_z A_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\ &\quad \text{Enough to know this}\end{aligned}$$

Moreover (if you know linear algebra...)

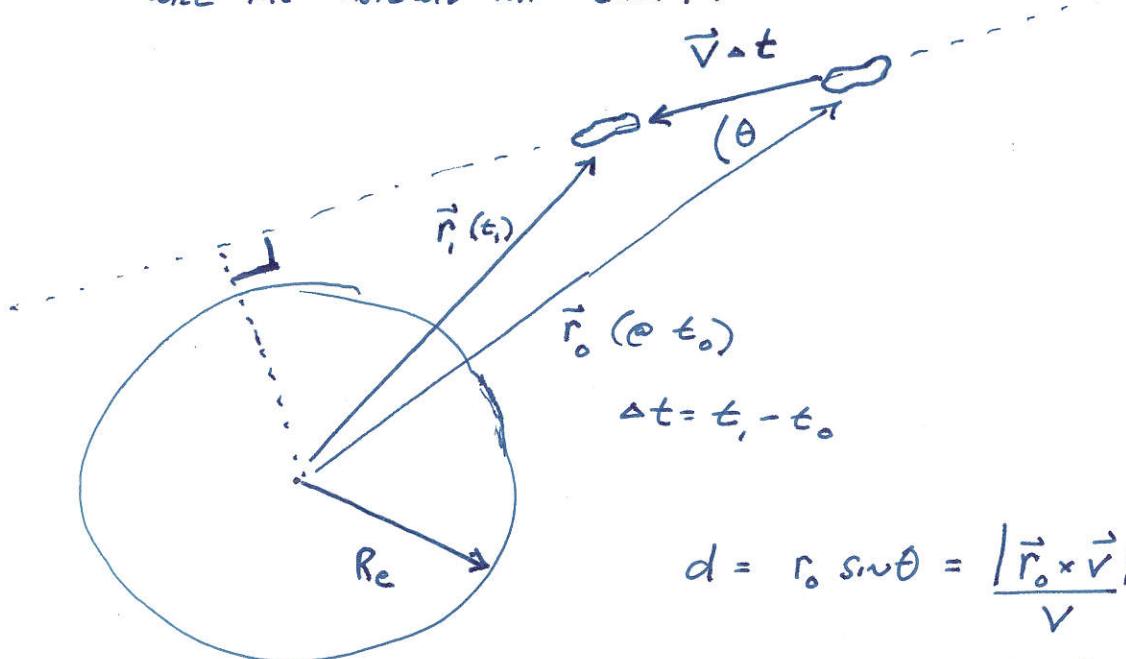
$$\vec{A} \times \vec{B} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix}$$

Also

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

"Triple vector product"

Ex Given measurements  $\vec{r}_0$  &  $\vec{r}_1$  of an asteroids position w.r.t. Earth, separated by time interval  $\Delta t$ , will the asteroid hit Earth?



$$d = r_0 \sin\theta = \frac{|\vec{r}_0 \times \vec{v}|}{v}$$

$$\text{and } \vec{v} \Delta t = \vec{r}_1 - \vec{r}_0$$

$$\begin{aligned} \text{so } d &= \frac{|\vec{r}_0 \times \left( \frac{\vec{r}_1 - \vec{r}_0}{\Delta t} \right)|}{\left| \frac{\vec{r}_1 - \vec{r}_0}{\Delta t} \right|} \\ &= \frac{|\vec{r}_0 \times \vec{r}_1|}{|\vec{r}_1 - \vec{r}_0|} ? < R_E \end{aligned}$$