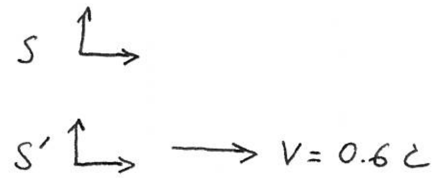
Physics 3210, Spring 2019 Homework #10

Due *in class* Thursday April 18^{th}

1. *K&K* Problem 12.7.
2. *K&K* Problem 12.9.
3. A lightweight pole 20 m long lies on the ground next to a barn 15 m long. An Olympic athlete picks up the pole, carries it far away, and runs with it towards the end of the barn at a speed of 0.8 c. Her friend remains at rest, standing by the barn door.
   (a) How long does the friend measure the pole to be, as it approaches the barn?
   (b) The barn door is initially open, and immediately after the runner and pole are entirely inside the barn, the friend shuts the door. How long after the door is shut does the front of the pole hit the other end of the barn, as measured by the friend?
   (c) In the reference frame of the runner, what is the length of the barn and the pole?
   (d) Does the runner believe that the pole is entirely inside the barn when it hits the end of the barn? Can you explain why, and the apparent contradiction between what is seen by the runner and what is seen by her friend?
4. *K&K* Problem 12.19.
5. A rocket flies between two planets that are one light-year apart. What should the rockets speed be so that the time elapsed on the captain's watch is one year?
6. A train of length 15 $c \cdot$sec moves at speed $3c/5$. How much time does it take to pass a person standing on the ground (as measured by that person)? Solve this by working in the frame of the person, and then again by working in the frame of the train.
7. A train of proper length L and speed $3c/5$ approaches a tunnel of length L. At the moment the front of the train enters the tunnel, a person leaves the front of the train and walks (briskly) toward the back. She arrives at the back of the train right when it (the back) leaves the tunnel.
   (a) How much time does this take in the ground frame?
   (b) What is the person's speed with respect to the ground?
   (c) How much time elapses on the person's watch?

K&K 12.7

1)



ORIGINS COINCIDE @ $t = t' = 0$

FIND COORDINATES IN S', GIVEN COORDINATES IN S

USE LORENTZ TRANSFORM EQUATIONS

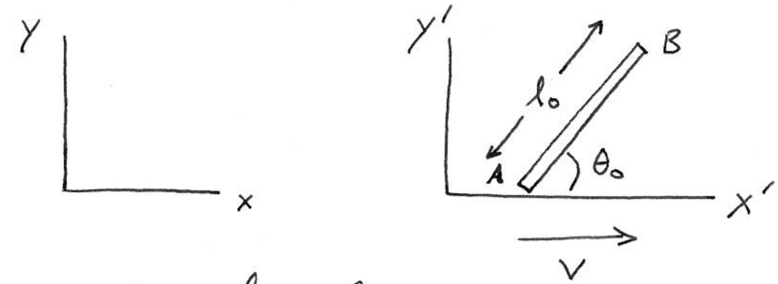
$$x' = \gamma(x - vt) \qquad \text{(EQ 12.3a)}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \text{(EQ 12.3b)}$$

WITH $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left(1 - 0.6^2\right)^{-1/2} = 1.25$

	x	t	x'	t'
a	4 m	0 s	5 m	-1×10^{-8} s
b	4 m	1 s	-2.25×10^{8} m	1.25 s
c	1.8×10^{8} m	1 s	0 m	0.8 s
d	10^{9} m	2 s	8×10^{8} m	0 s

2) K & K 12.9



We have $X'_{AB} = l_0 \cos\theta_0$

$Y'_{AB} = l_0 \sin\theta_0$

Now, length in unprimed frame is contracted in direction of motion

$$X_{AB} = \frac{1}{\gamma} X'_{AB}$$

But unchanged perpendicular to motion

$$Y_{AB} = Y'_{AB}$$

Let l, θ be length & angle in unprimed frame

$$\theta = \arctan\left(\frac{Y_{AB}}{X_{AB}}\right) = \arctan\left(\frac{l_0 \sin\theta_0}{l_0 \cos\theta_0 / \gamma}\right) = \boxed{\arctan\left(\gamma \tan\theta_0\right)}$$

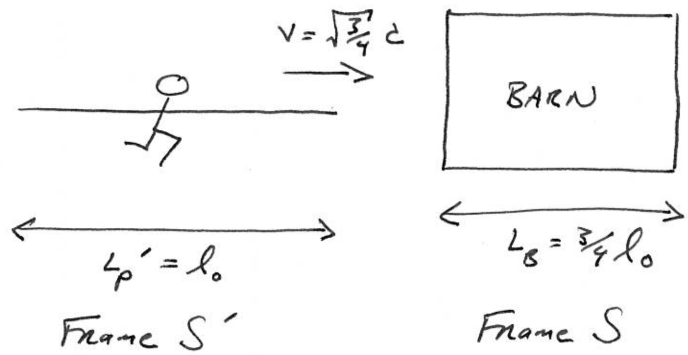
$$l = \sqrt{X_{AB}^2 + Y_{AB}^2} = \sqrt{\frac{l_0^2 \cos^2\theta_0}{\gamma^2} + l_0^2 \sin^2\theta_0} = \boxed{l_0 \sqrt{\frac{\cos^2\theta_0}{\gamma^2} + \sin^2\theta_0}}$$

3) K & K 12.16

Pole length l_0

Barn length $\frac{3}{4} l_0$

$V = \sqrt{\frac{3}{4}}\, c$



a) $L_p = \sqrt{1 - \frac{v^2}{c^2}}\; L_p'$ Length - contraction formula

$= \sqrt{1 - 3/4} \times l_0$

$$\boxed{L_p = l_0/2}$$

b) Pole vaulter sees BARN contracted by ½

$$L_B' = \frac{1}{2}\left(\frac{3}{4} l_0\right) = \frac{3}{8} L_0$$

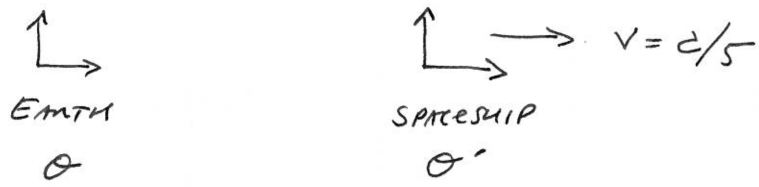
When front-of-pole reaches rear-of-barn, the rear-of-pole is still $\frac{5}{8} L_0$ outside (to left in frame) the barn.

c) Part (b) shows that front-of-pole will leave the barn before the rear enters, so both cannot be inside at the same time.

Note : Perhaps a more interesting version of this problem arises if the rear door is instead a rear wall. In that case, does the farmer end up closing the door or not?

Whether or not the door is ultimately closed cannot depend on the reference frame. So what happens?

4) K&K 12.19



IN THE EARTH'S FRAME, $d = vt$, WHERE d IS TWICE THE DISTANCE TO ALPHA CENTAURI.

$$t = \frac{d}{v} = \frac{2 \times 4.3 \text{ light-years}}{0.2 \times c} = 43 \text{ years},$$

EARTH TIME

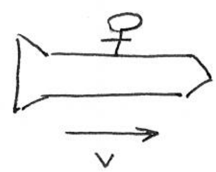
RELATE THIS TO SPACESHIP TIME VIA TIME DILATION EQUATION:

$$t = \frac{t'}{\sqrt{1 - v^2/c^2}} \; ; \; t' = 43 \cdot \sqrt{1 - 0.2^2} = 42.13 \text{ yrs}$$

SPACESHIP TIME

$$43 \text{ yrs} - 42.13 \text{ yrs} = 0.87 \text{ yrs} \sim \underline{10.4 \text{ MONTHS}}$$

5)



Frame O'

Frame O

$d = 1$ light-year

$$\frac{d}{v} = t = \frac{t'}{\sqrt{1 - v^2/c^2}}$$

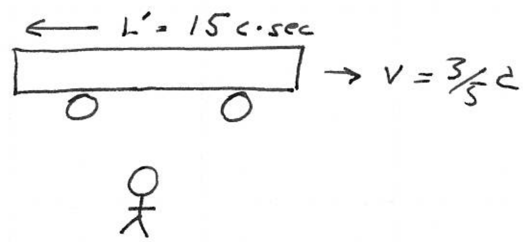
$$v = \left(\frac{t'^2}{d^2} + \frac{1}{c^2} \right)^{-1/2}$$

$t' = 1$ year

$d = c \times 1\,\text{yr}$

$$= \left(\frac{1}{c^2} + \frac{1}{c^2} \right)^{-1/2} = \boxed{\frac{c}{\sqrt{2}}}$$

6)



O'

O

Frame of Person

$$L = L'\sqrt{1 - v^2/c^2}\,, \quad t = L/v$$

$$t = \frac{L'}{v}\sqrt{1 - v^2/c^2} = 15\,c\cdot\text{sec} \cdot \frac{5}{3c} \cdot \sqrt{1 - (3/5)^2}$$

$$= \boxed{20 \text{ Seconds}}$$

Frame of Train

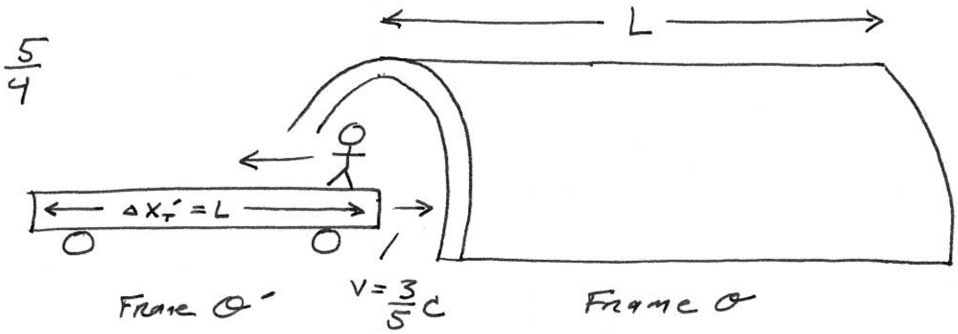
Person passes in $t' = L'/v = 15\,c\cdot\text{sec} \cdot \frac{5}{3c} = 25\,\text{sec}$

person's watch is slow, $t = \frac{t'}{\gamma} = 25\,\text{s} \times \frac{4}{5}$

$$= \boxed{20 \text{ seconds}}$$

7)

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{5}{4}$



⇒ WHEN FRONT OF TRAIN ENTERS TUNNEL SHE STARTS WALKING BACK, REACHING REAR OF TRAIN JUST AS REAR LEAVES TUNNEL.

a) Time in ground (O) frame?

TOTAL DISTANCE TRAVELED BY TRAIN $= L + \Delta X_T$

$$\Delta t = \frac{L + \Delta X_T}{v}$$

ΔX_T in ground frame $= \frac{L}{\gamma} = \frac{4}{5}L$, so $\Delta t = \frac{\frac{9}{5}L}{\frac{3}{5}c} = \boxed{\frac{3L}{c}}$

b) Person's SPEED WRT ground?

Person moves DISTANCE L WRT ground; $\frac{L}{\Delta t} = \frac{L}{3L/c} = \boxed{c/3}$

c) How much TIME PASSES ON PERSON'S WATCH?

$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2} = \frac{3L}{c} \cdot \sqrt{1 - 1/3^2} = \boxed{\frac{2\sqrt{2}\,L}{c}}$$