Physics 3210, Spring 2019 Homework #8

Due *in class* Thursday April 4^{th}

Fictitious Forces:

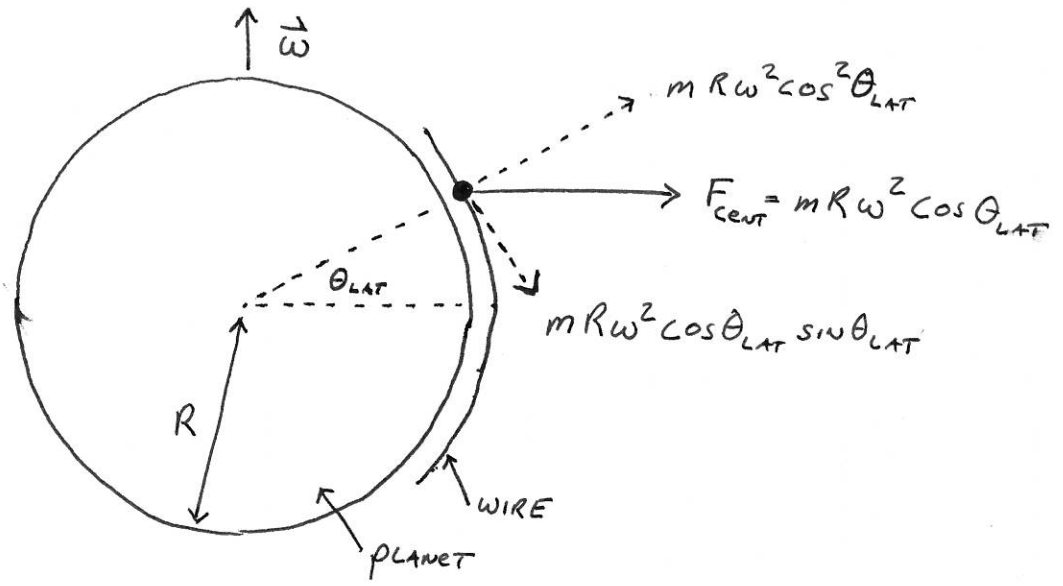
1. Consider a perfectly spherical rotating planet with an acceleration due to gravity $\vec{g}$ which is constant over the planet's surface. A bead lies on a frictionless wire that lies in the north–south direction across the equator. The wire takes the form of an arc of a circle; all points are the same distance from the center of the Earth. The bead is released from rest, a short distance from the equator. Because $\vec{g_{eff}}$ does not point directly toward the Earth's center, the bead will head toward the equator and undergo oscillatory motion. What is the frequency of these oscillations?

2. Hurricanes rotate in opposite directions in the Northern and Southern hemispheres due to the Coriolis force. A popular belief is that water swirls down the drain in opposite directions in the two hemispheres, for the same reason. Make a quantitative argument as to whether or not this belief is likely true.

3. A mass is dropped from a point directly above the equator. Consider the moment when the object has fallen a distance d. If we consider only the centrifugal force, then you can quickly show that the correction to g_{eff} at this point (relative to the release point) is an increase by $\omega^2 d$. There is, however, also a second-order Coriolis effect. What is the sum of these corrections? How do these effects compare to the variation of g with height?

4. *K&K* Problem 9.12.

1)



ACCELERATION ALONG THE WIRE:

$$F = ma$$

$$-mR\omega^2\cos\theta_{LAT}\sin\theta_{LAT} = mR\ddot{\theta}$$

$$\ddot{\theta} + \omega^2\cos\theta\sin\theta = 0$$

Small angle approximation, $\cos\theta\sin\theta \approx 1\cdot\theta \approx \theta$

$$\ddot{\theta} + \omega^2\theta = 0$$

→ SIMPLE HARMONIC OSCILLATION, FREQUENCY IS THE SAME AS ROTATIONAL FREQUENCY OF PLANET. ∇

2) $\vec{F}_{\text{coriolis}} = -2m\vec{\omega} \times \vec{v}$

FOR A BALLPARK ESTIMATE, TAKE $F_{cor} = 2m\omega v = ma$

A particle might be deflected $d = \frac{1}{2}at^2$ in time t

$= \omega v t^2$

Now guess some numbers ...

$$\omega_{earth} = \frac{2\pi}{24\,hrs} = 7.3\times10^{-5}/s$$

$v \approx 0.1\,\frac{m}{s}$ SPEED OF WATER IN BATHTUB

$t \approx 10\,sec$ TIME FOR WATER TO REACH DRAIN

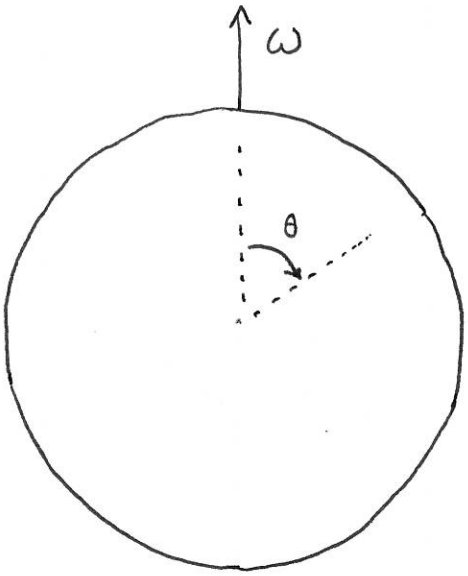
$$d \approx \left(7.3\times10^{-5}/s\right)\cdot\left(0.1\,\tfrac{m}{s}\right)\left(10\,sec\right)^2$$

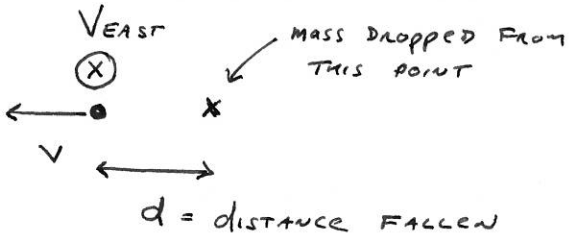
≈ 0.7 MICROMETERS.

$\Rightarrow$ THIS IS SMALL COMPARED TO, e.g. THE SIZE OF A TYPICAL DRAIN. ALSO COMPARED TO TURBULENCE IN A BATHTUB.

$\therefore$ THE CORIOLIS EFFECT WOULD BE <u>VERY DIFFICULT TO SEE</u> IN TYPICAL WATER-DOWN-THE-DRAIN SITUATIONS.

3)





⇒ We have the in-class result that $g_{eff} = g - \omega^2 r \sin^2\theta$

$= g - \omega^2 r$ AT EQUATOR

So $\Delta g_{eff} = g_{eff,final} - g_{eff,initial} = \boxed{\omega^2 d}$, i.e. "gravity" increases as mass falls.

⇒ While falling, mass will experience a Coriolis force of magnitude $2m\omega v = 2m\omega g t$ towards the <u>EAST</u> (into the page, in Figure)

where V is <u>downward</u> speed.

⇒ Thus mass will acquire an <u>eastward</u> component to velocity

$$V_E = \int_0^t \left(\frac{F_{cor}}{m}\right) dt = \int_0^t \left(\frac{2m\omega g t}{m}\right) dt = \omega g t^2$$

Since distance fallen $d = \frac{1}{2} g t^2$

$$V_E = 2\omega d$$

⟹

3) Continued . . .

⇒ The existing velocity component will now produce an "upward" <u>second-order</u> Coriolis force!

$$F_{up} = 2m\omega V_E = 4m\omega^2 d$$

Therefore $\Delta g_{eff} = \omega^2 d - 4\omega^2 d$

↑ increase due to centrifugal ↑ decrease due to 2nd-order Coriolis

$$\boxed{\Delta g_{eff} = -3\omega^2 d}$$

⇒ Now compare with variation of g with height

$$\Delta g_{height} = \frac{GM}{(r-d)^2} - \frac{GM}{r^2}$$

$$= \frac{2GM\left(\frac{d}{r}\right) - Gm\left(\frac{d}{r}\right)^2}{(r-d)^2}$$

In limit $d \ll r \approx \frac{2GM}{r^3} d$

Compare: $\omega^2 = (7.3\times10^{-5})^2 = \underline{5\times10^{-9}\ /s^2}$

$$\frac{2GM}{r^3} = \frac{2\times(6.7\times10^{-11})(6.0\times10^{24})}{(6.4\times10^6)^3} = \underline{3.7\times10^{-6}\ /s^2}$$

↳ The change in g due to variation in the force with height is about 1000× larger than centrifugal and Coriolis effects.

4) K&K 9.12

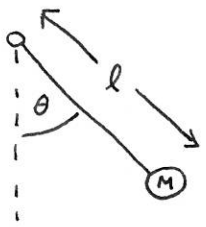
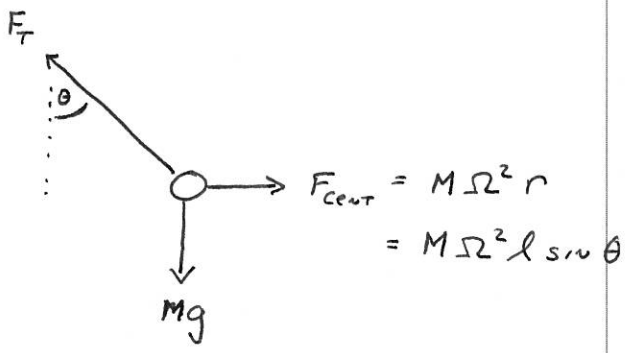


DIAGRAM OF PENDULUM IN ROTATING FRAME.



FREE-BODY DIAGRAM.

(NOTE THAT CORIOLIS FORCE IS ⊥ TO PLANE OF MOTION.
BUT PENDULUM IS CONFINED TO PLANE, SO CORIOLIS HAS NO EFFECT.)

⇒ SUM TORQUES ABOUT AXIS OF PENDULUM...

$$\sum \tau = F_{cent} \cdot l\cos\theta - Mgl\sin\theta = I\alpha = Ml^2\ddot{\theta}$$

$$M\Omega^2 l^2 \cos\theta\sin\theta - Mgl\sin\theta = Ml^2\ddot{\theta}$$

SMALL ANGLES, $\cos\theta\sin\theta \sim \sin\theta \sim \theta$

$$(M\Omega^2 l^2 - Mgl)\theta = Ml^2\ddot{\theta}$$

$$\ddot{\theta} + \left(\frac{g}{l} - \Omega^2\right)\theta = 0$$

2 CASES: (i) $\Omega^2 > \frac{g}{l}$ SOLN TO DIFFERENTIAL EQUATION IS EXPONENTIAL,
∴ AMPLITUDE NOT SMALL

(ii) $\Omega^2 < \frac{g}{l}$ SIMPLE HARMONIC OSCILLATION WITH

$$\boxed{\omega = \sqrt{\frac{g}{l} - \Omega^2}}$$