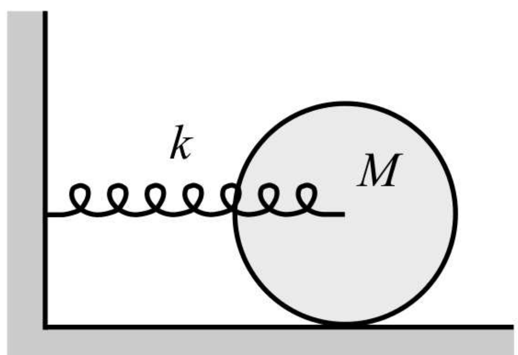
Physics 3210, Spring 2019 — Homework #7

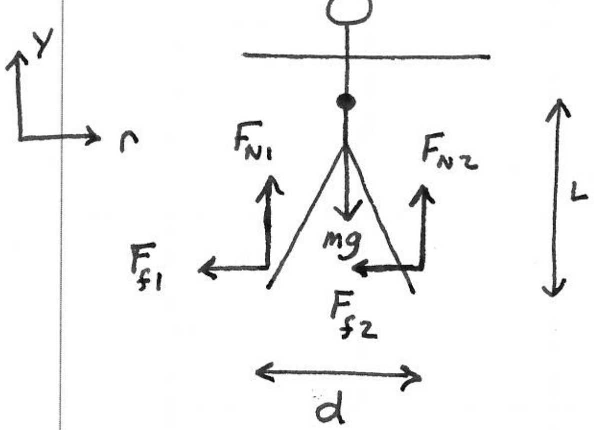
Due *in class* Thursday March 7^{th}

Rotational Dynamics:

1. *K&K* Problem 7.6.
2. *K&K* Problem 7.7.
3. *K&K* Problem 7.11.
4. *K&K* Problem 7.13.
5. *K&K* Problem 7.18.
6. *K&K* Problem 7.20.
7. *K&K* Problem 7.21.
8. The axle of a solid cylinder of mass m and radius r is connected to a spring with spring constant k, as shown in the figure. If the cylinder rolls without slipping, what is the frequency of the oscillations?



1) K & K 7.6



THE FORCE ON EACH FOOT WILL HAVE A HORIZONTAL FRICTION COMPONENT AND A VERTICAL NORMAL COMPONENT.

RECALL THAT IT IS THE NORMAL COMPONENT THAT WE ASSOCIATE WITH "WEIGHT".

(i) $$\sum F_y = F_{N1} + F_{N2} - Mg = 0$$

(ii) $$\sum F_r = -F_{f1} - F_{f2} = -\frac{mv^2}{R}$$

(iii) $$\sum \tau = -F_{N1}\frac{d}{2} + F_{N2}\frac{d}{2} - F_{f1}L - F_{f2}L = 0$$

From (ii) we have $F_{f1} + F_{f2} = \frac{mv^2}{R}$, plug into (iii) to eliminate friction...

$$F_{N2} - F_{N1} = \frac{2mv^2L}{dR}$$

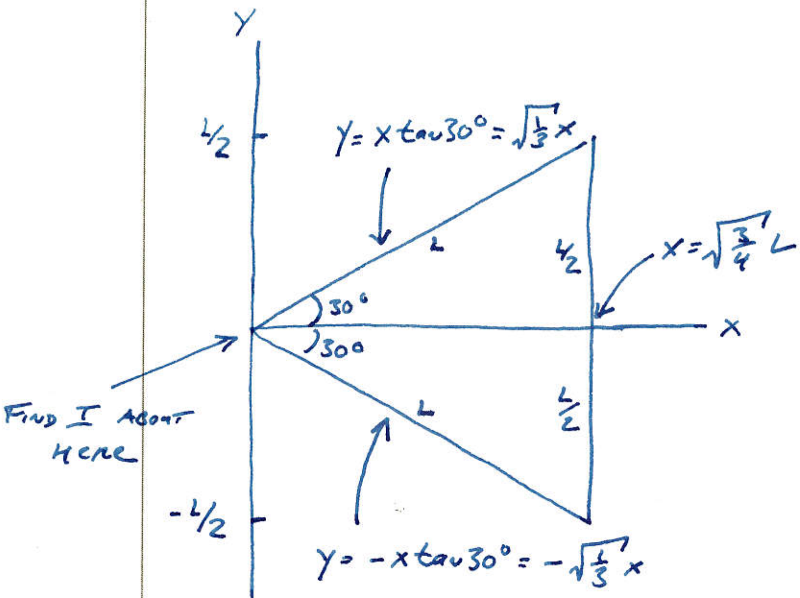
& from (i)

$$F_{N2} + F_{N1} = mg$$

So...

$$\boxed{\begin{aligned} F_{N1} &= \frac{mg}{2} - \frac{mv^2L}{dR} \\ F_{N2} &= \frac{mg}{2} + \frac{mv^2L}{dR} \end{aligned}}$$

2) K&K 7.7



$$\sigma = \frac{\text{Mass}}{\text{Area}} = \frac{M}{\frac{1}{2} b \times h} = \frac{M}{\frac{1}{2} L \cdot \sqrt{\frac{3}{4}} L}$$

$$\sigma = \frac{4M}{\sqrt{3} L^2}$$

$$I_{\text{origin}} = \int_{x=0}^{\sqrt{\frac{3}{4}}L} \int_{y=-\sqrt{\frac{1}{2}}x}^{\sqrt{\frac{1}{3}}x} (x^2 + y^2)\,\sigma\, dx\, dy$$

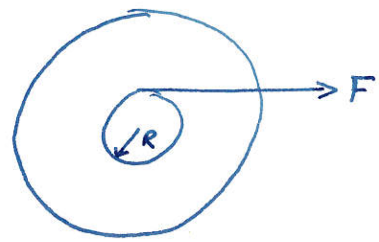
$$= \sigma \int_{x=0}^{\sqrt{\frac{3}{4}}L} \left[x^2 y + \frac{y^3}{3}\right]_{-\sqrt{\frac{1}{3}}x}^{\sqrt{\frac{1}{3}}x} dx = \sigma \int_{x=0}^{\sqrt{\frac{3}{4}}L} \frac{20}{\sqrt{3}\, 9} x^3\, dx$$

$$= \frac{\sigma}{\sqrt{3}} \cdot \frac{20}{9} \cdot \frac{1}{4} \left(\frac{\sqrt{3}}{2}\right)^4 \cdot L^4$$

$$= \frac{4M}{\sqrt{3} L^2} \cdot \frac{1}{\sqrt{3}} \cdot \frac{20}{9} \cdot \frac{1}{4} \cdot \frac{9}{16} L^4$$

$$\boxed{I_{\text{origin}} = \frac{5}{12} M L^2}$$

3) KdK 7.11



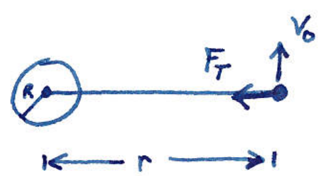
WORK DONE = CHANGE IN K.E.

$$F \cdot L = \frac{1}{2} I \omega_0^2$$

$$\boxed{I = \frac{2FL}{\omega_0^2}}$$

4) KdK 7.13

case a)



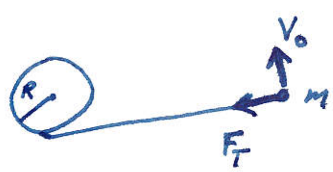
$\Rightarrow$ FORCE IS CENTRAL $\rightarrow$ ANGULAR MOMENTUM CONSERVED

$$L = m v_f R = m v_0 r = \text{CONSTANT}$$

$$\boxed{V_f = V_0 \frac{r}{R}}$$

$\Rightarrow$ NOTE THAT $\vec{p}$ AND E ARE NOT CONSERVED, as external force is acting on m.

case b)



$\Rightarrow$ FORCE NOT CENTRAL, SO ANGULAR MOMENTUM NOT CONSERVED

$\Rightarrow$ MOMENTUM IS NOT CONSERVED,

$$\vec{F}_T = \frac{d\vec{p}}{dt}$$

$\Rightarrow$ ENERGY IS CONSERVED, SINCE $\vec{F}_T \perp \vec{v}$ IT DOES NO WORK

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2$$

$$\boxed{V_f = V_0}$$

5) K&K 7.18

We have the in-class result that $T = 2\pi\sqrt{\frac{I}{mgR_{cm}}}$ for physical pendulum

Calculate moment-of-inertia of rod & disk pendulum...

$$I = I_{ROD} + I_{DISK} = I_{ROD}^{END} + \left[M_{DISK} R_{cm,\,DISK}^2 + I_{DISK}^{CENTER}\right]$$

$$= \frac{1}{3} m l^2 + M l^2 + \frac{1}{2} M R^2$$

Calculate distance of center of mass from hinge

$$R_{cm} = \frac{\frac{ml}{2} + Ml}{m+M}$$

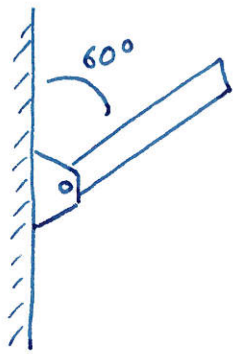
Combining

$$\boxed{T = 2\pi\sqrt{\frac{\frac{1}{3} m l^2 + M l^2 + \frac{1}{2} M R^2}{\frac{mgl}{2} + Mgl}}}$$

⇒ If disk is mounted by frictionless bearing, it won't rotate. (No torque!) So we should omit the term due to the moment of the disk around its center, $\frac{1}{2}MR^2$

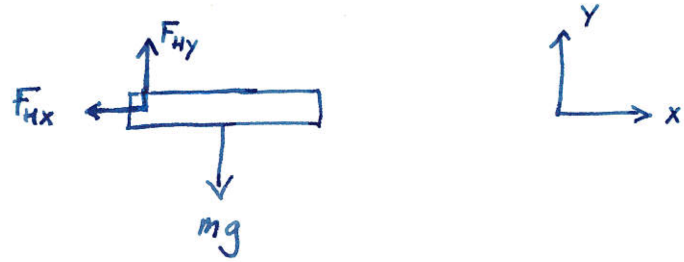
$$T = 2\pi\sqrt{\frac{\frac{1}{3} m l + M l}{\frac{mg}{2} + Mg}}$$

6) K&K 7.20



released $\theta = 60°$

FBD



Horizontal $\theta = 90°$

⇒ 1st, sum torques @ $\theta = 90°$

$$\Sigma \tau = \frac{mg\ell}{2} = I\alpha = \frac{1}{3} m \ell^2 \ddot{\theta}_{90}$$

$$\boxed{\ddot{\theta}_{90} = \frac{3}{2}\frac{g}{\ell}}$$

⇒ Sum Forces

$$\Sigma F_x = -F_{Hx} = \frac{mv^2}{r} = \frac{mv^2}{\ell/2} = m\frac{\ell}{2}\dot{\theta}_{90}^2$$

$$\boxed{F_{Hx} = -\frac{m\ell}{2}\dot{\theta}_{90}^2}$$

$$\Sigma F_y = F_{Hy} - mg = ma_y = -m\frac{\ell}{2}\ddot{\theta}_{90} = -\frac{m\ell}{2}\cdot\frac{3}{2}\frac{g}{\ell}$$

$$\boxed{F_{Hy} = \frac{1}{4}mg}$$

⇒ get $\dot{\theta}$ from energy conservation $\quad mg\frac{\ell}{2}\sin 30° = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\dot{\theta}_{90}^2$

$$\boxed{\dot{\theta}_{90}^2 = \frac{3}{2}\frac{g}{\ell}}$$

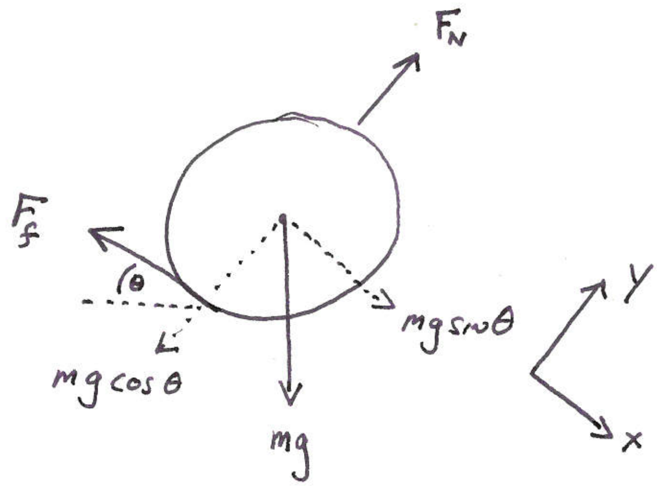
So $\vec{F}_H = \left(-\frac{3}{4}mg\,\hat{x} + \frac{1}{4}mg\,\hat{y}\right)$

Note: Book asks for force on hinge/pivot, which is $-\vec{F}_H$

$$\boxed{\vec{F}_{\text{on hinge}} = \left(\frac{3}{4}mg\,\hat{x} - \frac{1}{4}mg\,\hat{y}\right)}$$

7) K&K 7.21

FBD:



$$\Sigma F_x = mg\sin\theta - F_f = ma_x \quad (i)$$

$$\Sigma F_y = F_N - mg\cos\theta = 0 \quad (ii)$$

$$\Sigma \tau = -F_f \cdot R = I\alpha = -\underbrace{\left(\tfrac{1}{2}MR^2\right)}_{\text{I OF DISK}} \cdot \underbrace{\left(\frac{a_x}{R}\right)}_{\text{ROLLING W/OUT SLIPPING.}}$$

$$F_f = \tfrac{1}{2}Ma_x \quad (iii)$$

Combine (i) and (iii) $\quad mg\sin\theta - \frac{1}{2}ma_x = ma_x$

$$a_x = \tfrac{2}{3}g\sin\theta$$

For rolling w/out slipping

$$\tfrac{1}{2}ma_x = F_f \leq \mu_s F_N = \mu_s mg\cos\theta$$

or

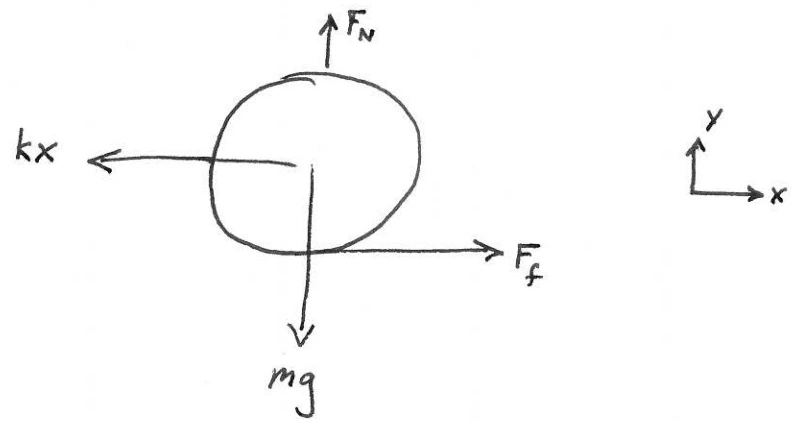
$$\boxed{\tan\theta \leq 3\mu_s}$$

Is rolling-without-slipping condition.

8)

Method I: Forces & Torques

FBD:



At this particular instant, x = positive
v = negative
a = negative

Note that, in general, F_f opposes acceleration

$$\sum \tau = -F_f r = \frac{1}{2} m r^2 \alpha = \frac{1}{2} m r^2 \frac{a}{r}$$

$$F_f = -\frac{1}{2} m a$$

$$\sum F_x = F_f - kx = ma$$

$$-\frac{1}{2} m\ddot{x} - kx = m\ddot{x}$$

$$\ddot{x} + \left(\frac{2}{3}\frac{k}{m}\right)x = 0$$

Simple Harmonic Oscillator with $\boxed{\omega = \sqrt{\frac{2}{3}\frac{k}{m}}}$

$\longrightarrow$

Method II : Lagrange

$$K = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{x}}{r}\right)^2$$

$$= \frac{3}{4} m\dot{x}^2$$

$$U = \frac{1}{2}kx^2$$

$$L = K - U = \frac{3}{4} m\dot{x}^2 - \frac{1}{2}kx^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{3}{2} m\ddot{x} + kx = 0$$

$$\ddot{x} + \left(\frac{2}{3}\frac{k}{m}\right)x = 0 \quad \checkmark$$

Method III: Energy Conservation

$$\frac{d}{dt}\left[\frac{3}{4} m\dot{x}^2 + \frac{1}{2} kx^2 = C\right]$$

$$\frac{3}{2} m\cancel{\dot{x}}\ddot{x} + kx\cancel{\dot{x}} = 0$$

$$\ddot{x} + \left(\frac{2}{3}\frac{k}{m}\right)x = 0 \quad \checkmark$$