Physics 3210, Spring 2019 Homework #6

Due *in class* Thursday February 28^{th}

Lagrangian Mechanics:

1. Two blocks, both of mass M, are connected with a spring with spring constant k. They slide in the x-direction on a frictionless surface.

   (a) Use the Lagrangian formalism to find coupled differential equations for the positions x_1 and x_2 of the two blocks.

   (b) Use these equations to show that the center-of-mass of the system moves with constant velocity.

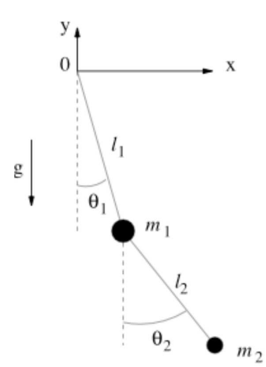
   (c) Find the period of oscillation of the two masses with respect to each other.

2. Consider the *double pendulum* in the figure below, consisting of two bobs of mass m_1 and m_2 at the ends of massless rods of length l_1 and l_2 in which the second rod swings from the first mass.

   (a) Derive (but do not solve) the equations of motion for the two bobs in terms of their coordinates θ_1 and θ_2. You may find the arithmetic easier if you make use of the trigonometric identity

$$\cos A \cos B + \sin A \sin B = \cos (A - B)$$

   (b) Find the simplified version of these equations for the case in which θ_1 and θ_2 are both small angles.

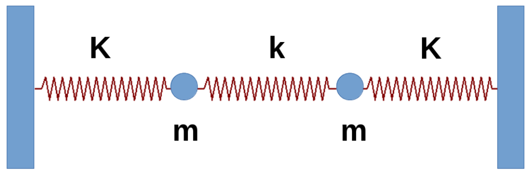


Additional problems on the back of this page.

Normal Modes of Oscillation:

3. *K&K* Problem 6.3.

4. See the figure below. Three springs and two equal masses lie between two walls. The spring constant K of the outer springs is larger than the spring constant k of the inner spring. Let x_1 and x_2 be the positions of the left and right masses, respectively, relative to their equilibrium positions.



(a) Find general oscillatory solutions for $x_1(t)$ and $x_2(t)$.

(b) Assume $K/m = 1.0/\mathrm{s}^2$ and $k = 0.2\,K$. Plot x_1 and x_2 versus time for the normal modes of the system. Assume the equilibrium distance between the two masses is 1.0 m.

(c) Now, find the particular solutions for x_1 and x_2, for the initial conditions:

$$\begin{aligned} x_1(0) &= 0.0 \text{ m} \\ \dot{x}_1(0) &= 0.0 \text{ m/s} \\ x_2(0) &= 0.4 \text{ m} \\ \dot{x}_2(0) &= 0.0 \text{ m/s} \end{aligned}$$

Plot these solutions for t in the range $0 - 100$ s.

(Note: If you don't have access to a graphing program on your computer, have a look at online plotting tools such as **WolframAlpha** and **fooplot.com**.)