Physics 3210, Spring 2019 Homework #5

Due *in class* Thursday February 21^{st}

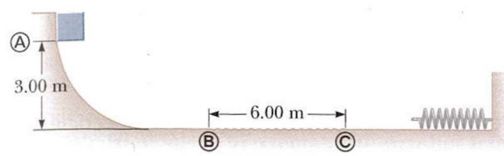
1. Using the general solution for the position of a mass on the end of a spring:

$$x(t) = A\cos(\omega t + \phi)$$

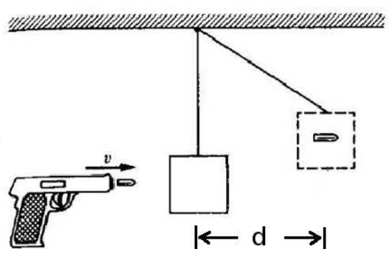
verify that the total mechanical energy (kinetic plus potential) is a constant of the motion.

2. A roller coaster car starts at rest and coasts down a frictionless track. It encounters a vertical loop of radius R. How much higher than the top of the loop must the car start if it is to remain in contact with the track at all times?

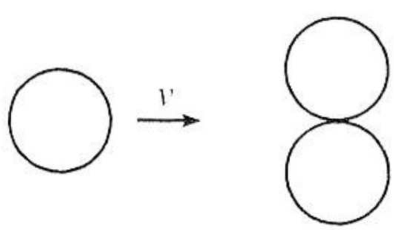
3. A 12.0 kg block is released from rest at point A in the figure. The track is frictionless except for the portion between points B and C, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2,400 N/m, and compresses the spring 0.250 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between points B and C.



4. A bullet with mass $m = 0.004$ kg is fired into a ballistic pendulum with length $L = 0.75$ m and mass $M = 3.0$ kg as shown in the figure on the left below. The pendulum's maximum horizontal displacement $d = 0.40$ m. What is the initial speed v of the bullet?



5. A pool ball with initial speed v is aimed right between two other pool balls, as shown in the figure. If the two right balls leave the elastic collision with equal speeds, find the final velocities (magnitude and direction) of all three balls.



1)

$$x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$V(t) = -\omega A \sin(\omega t + \phi)$$

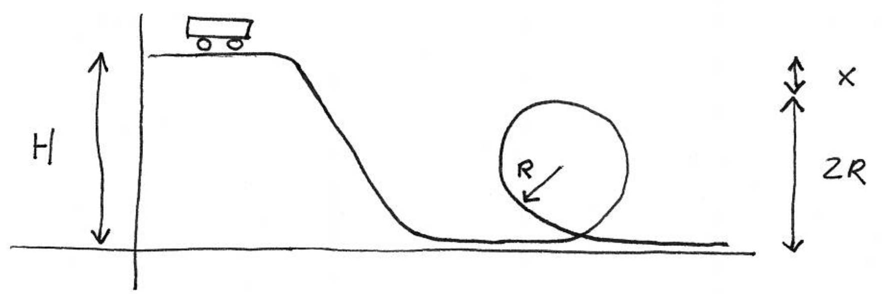
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi) + \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

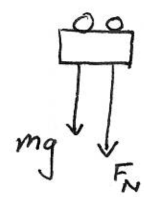
$$= \frac{1}{2}kA^2\left[\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right]$$

$$= \frac{1}{2}kA^2 = \text{CONSTANT} \checkmark$$

2)



FBD OF CAR
AT TOP OF LOOP:



$$\sum F_r = -mg - F_N = \frac{-mv^2}{R}$$

IF CAR JUST REMAINS IN CONTACT

$$F_N = 0$$

$$\hookrightarrow v^2 = Rg \quad (i)$$

Apply energy conservation:

$$mgx = \frac{1}{2}mv^2$$

$$v^2 = 2gx \quad (ii)$$

Combine (i) & (ii)

$$\boxed{X = R/2}$$

3) Apply Energy conservation

$$mg\,y_A = \frac{1}{2} m v_B^2$$

$$\frac{1}{2} m v_c^2 = \frac{1}{2} k x_f^2$$

Work-Energy relation for Friction Force

$$\int_B^c F\,dx = -F_f \cdot x_{BC} = \Delta K = \frac{1}{2} m v_c^2 - \frac{1}{2} m v_B^2$$

$$F_f = \mu_k mg$$

Combining results above

$$-\mu_k mg\, x_{BC} = \frac{1}{2} m v_c^2 - \frac{1}{2} m v_B^2 = \frac{1}{2} k x_f^2 - mg\,y_A$$

$$\mu_k = \frac{mg\,y_A - \frac{1}{2} k x_f^2}{mg\,x_{BC}}$$

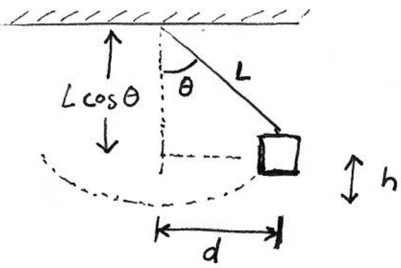
$m = 12\,kg$
$g = 9.81\,m/s^2$
$y_A = 3.0\,m$
$k = 2{,}400\,N/m$
$x_f = 0.250\,m$
$x_{BC} = 6.00\,m$

$$= \frac{(12\,kg)(9.81\,m/s^2)(3.0\,m) - 0.5(2{,}400\,N/m)(0.250\,m)^2}{(12\,kg)(9.81\,m/s^2)(6.00\,m)}$$

$$\boxed{\mu_k = 0.394}$$

4)

AT BULLET - PENDULUM IMPACT,
MOMENTUM CONSERVATION GIVES:



$$m v = (m + M) v_i$$

THEN APPLY MECHANICAL ENERGY CONSERVATION AS BULLET/BOB SWINGS UPWARD.

$$\frac{1}{2}(m+M) v_i^2 = (m+M) g h$$

$$v_i = \sqrt{2gh} = \sqrt{2g\left(L - \sqrt{L^2 - d^2}\right)}$$

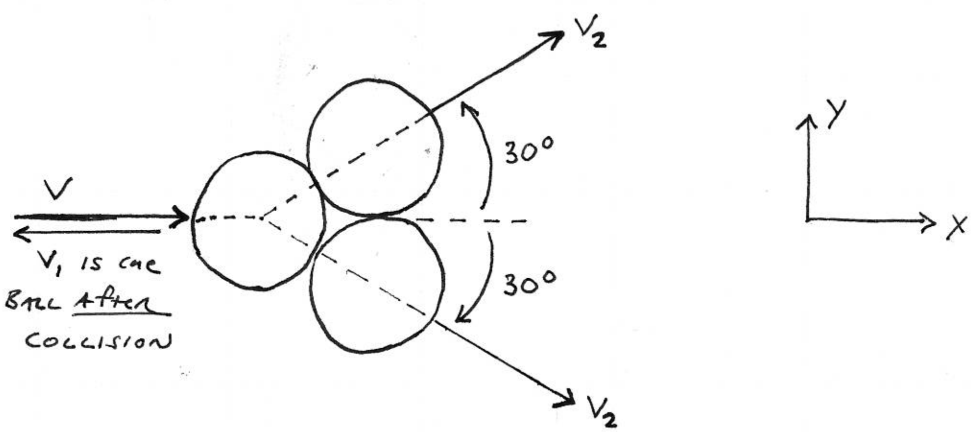
So,

$$v = \frac{(m+M)}{m}\sqrt{2g\left(L - \sqrt{L^2-d^2}\right)}$$

$$= \frac{(0.004\ kg + 3.0\ kg)}{(0.004\ kg)}\sqrt{2 \cdot 9.81 \, m/s^2 \cdot \left(0.75\,m - \sqrt{(0.75\,m)^2 - (0.4\,m)^2}\right)}$$

$$\boxed{v = 1131\ m/s}$$

5)



$\Rightarrow$ THIS IS AN <u>ELASTIC COLLISION</u>, SO BOTH <u>MOMENTUM</u> AND <u>ENERGY CONSERVATION</u> APPLY HERE

$$mv = mv_1 + 2m\,v_2 \cos 30°$$
$$= mv_1 + 2m\,v_2 \cdot \sqrt{\tfrac{3}{4}}$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_1^2 + 2 \cdot \tfrac{1}{2}m v_2^2$$

DIVIDE OUT ALL THE M'S TO OBTAIN

$$V = V_1 + \sqrt{3}\,V_2$$
$$V^2 = V_1^2 + 2V_2^2$$

ARITHMETIC

$$\boxed{V_1 = -V/5 \quad (\text{ie to left}) \qquad V_2 = \frac{2\sqrt{3}}{5}V}$$