Physics 3210, Spring 2019 Homework #2

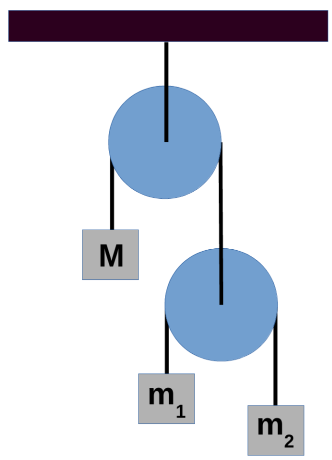
Due *in class* Thursday January 24^{th}

Newton's Laws:

1. *K&K* Problem 2.2.

2. *K&K* Problem 2.3.

3. A two-pulley system is set up as shown in the figure below. The first (massless) pulley supports mass M and a second massless pulley. The second pulley supports masses m_1 and m_2. What should mass M be, in terms of m_1 and m_2, such that it doesn't move?



Rotational Problems:

4. *K&K* Problem 2.6.

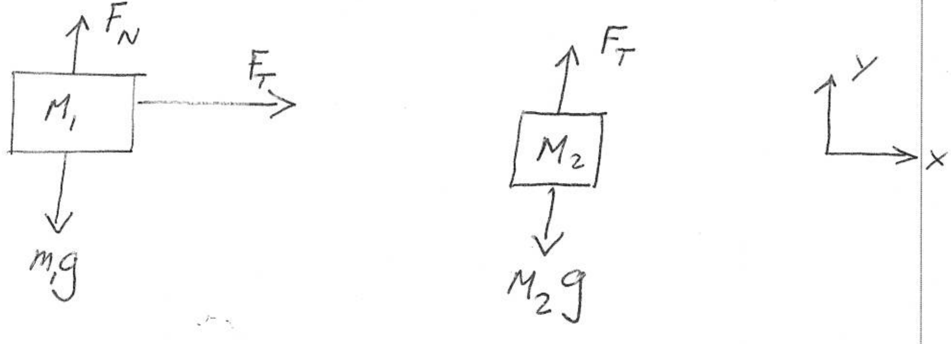
5. *K&K* Problem 2.15.

Units:

6. *K&K* Problem 2.16.

1) K&K 2.2

FBD



Block 1: $\sum F_x = F_T = m_1 a_{1x}$

$$\sum F_y = F_N - m_1 g = 0$$

Block 2: $\sum F_y = F_T - m_2 g = m_2 a_{2y}$

Now $a_{2y} = -a_{1x} = -a$

$$F_T = m_1 a$$

$$F_T = m_2 g - m_2 a$$

Eliminate F_T

$$m_1 a = m_2 g - m_2 a$$

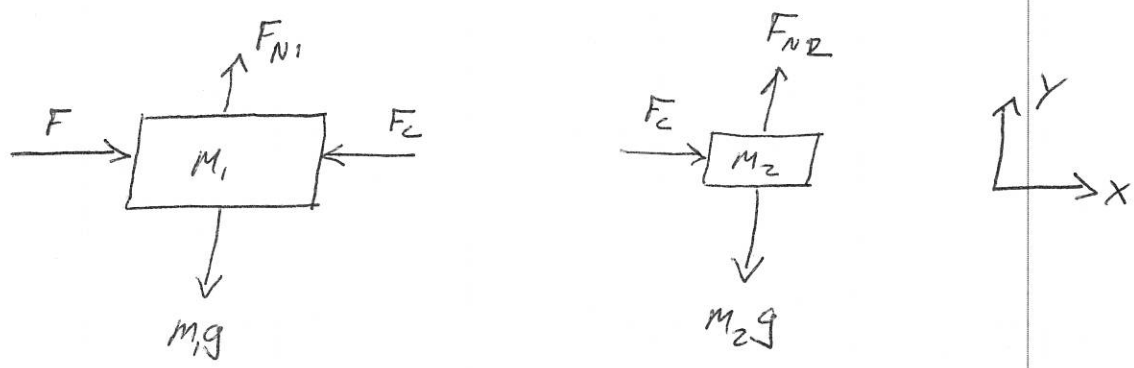
$$a = \frac{m_2 g}{(m_1 + m_2)}$$

In time t, block m_1 will travel

$$x_1(t) = \frac{1}{2} a t^2 = \boxed{\frac{m_2 g}{2(m_1 + m_2)} t^2}$$

2) <u>K&K 2.3</u>

<u>FBD</u>



Block 1 $$\sum F_x = F - F_c = m_1 a_{1x}$$
$$\sum F_y = F_{N1} - m_1 g = 0$$

Block 2 $$\sum F_x = F_c = m_2 a_{2x}$$
$$\sum F_y = F_{N2} - m_2 g = 0$$

$\Rightarrow$ Blocks in contact, so $a_{1x} = a_{2x} = a$

$$F - F_c = m_1 a$$
$$F_c = m_2 a$$

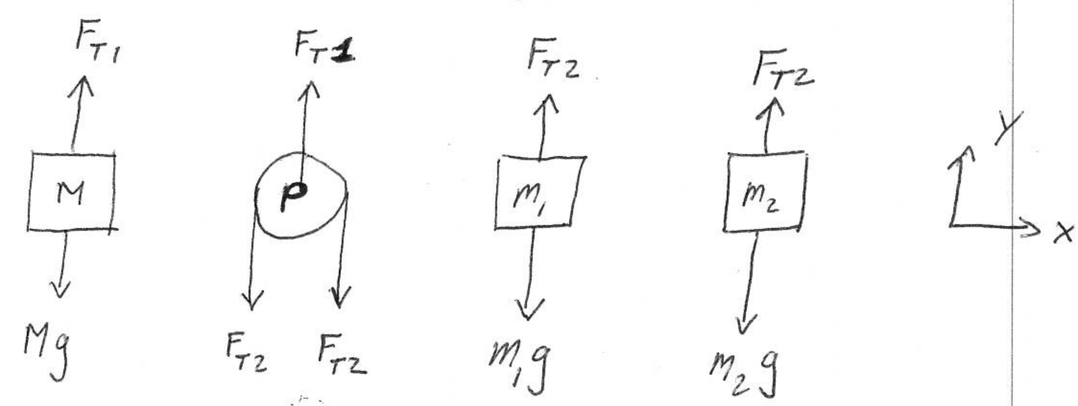
$\Rightarrow$ Eliminate a

$$\frac{F - F_c}{m_1} = \frac{F_c}{m_2}$$

$$F_c = \left(\frac{m_2}{m_1 + m_2}\right) F = \left(\frac{1\,\text{kg}}{2\,\text{kg} + 1\,\text{kg}}\right)(3\,\text{N})$$

$$\boxed{F_c = 1\,\text{N}}$$

3) FBD



Sum Forces (y-direction ONLY)

Block M: $\sum F_y = F_{T1} - Mg = Ma_M = 0$ (IN STATEMENT OF PROBLEM)

Pulley $\sum F_y = F_{T1} - 2F_{T2} = m_p a_p = 0$

Block m_1 $\sum F_y = F_{T2} - m_1 g = m_1 a_1$

Block m_2 $\sum F_y = F_{T2} - m_2 g = m_2 a_2$

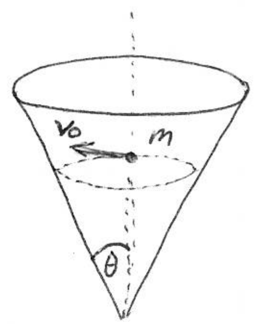
$\Rightarrow \quad a_2 = -a_1 = -a$

$$\left.\begin{aligned} &(i) \quad F_{T1} = Mg \\ &(ii) \quad F_{T1} = 2F_{T2} \end{aligned}\right\} \quad F_{T2} = \frac{Mg}{2}$$

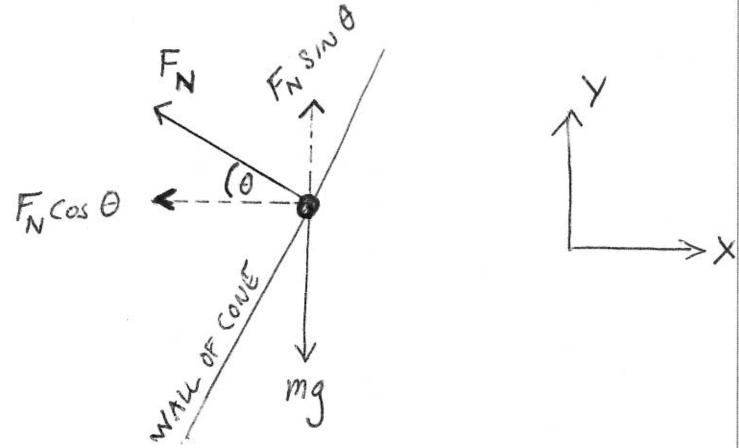
$$\left.\begin{aligned} &(iii) \quad F_{T2} = m_1 g + m_1 a \\ &(iv) \quad F_{T2} = m_2 g - m_2 a \end{aligned}\right\} \quad \underbrace{\begin{aligned} \frac{Mg}{2} &= m_1 g + m_1 a \\ \frac{Mg}{2} &= m_2 g - m_2 a \end{aligned}}_{\text{eliminate } a,\text{ solve for } M}$$

$$\boxed{M = \frac{4m_1m_2}{m_1+m_2}}$$

4) K&K 2.6



Force Diagram (velocity is into page)



(i) $\sum F_x = -F_N \cos\theta = m a_x = -m r \omega^2 = -\dfrac{m v_0^2}{r}$

(ii) $\sum F_y = F_N \sin\theta - mg = m a_y = 0$; $F_N = \dfrac{mg}{\sin\theta}$

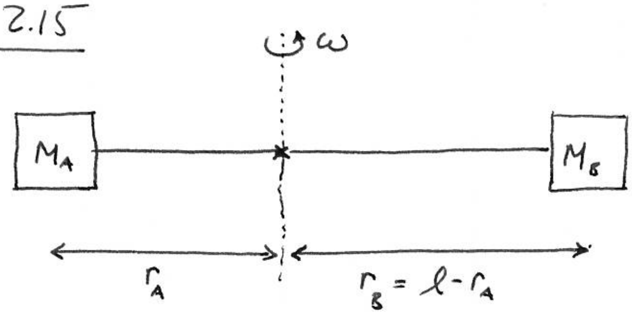
Insert result of (ii) into (i)

$$\frac{mg}{\sin\theta} \cos\theta = \frac{m v_0^2}{r}$$

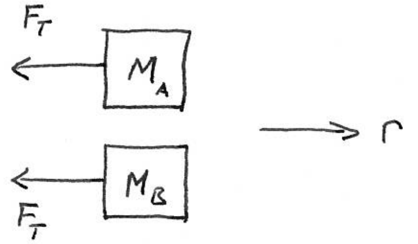
$$\boxed{r = \frac{v_0^2 \tan\theta}{g}}$$

5) <u>K & K 2.15</u>

@ $t=0$



<u>FBD</u>



NOTE THAT IN BOTH CASES F_T IS IN NEGATIVE $\hat{r}$ direction.

$$\sum F_A = -F_T = M_A\left(\ddot{r}_A - r_A\omega^2\right)$$

$$\sum F_B = -F_T = M_B\left(\ddot{r}_B - r_B\omega^2\right)$$

CONSTRAINTS: $r_B = l - r_A$, $\ddot{r}_B = -\ddot{r}_A$

ELIMINATING F_T:

$$M_A\ddot{r}_A - M_A r_A\omega^2 = m_B\ddot{r}_B - M_B r_B\omega^2$$

$$= -m_B\ddot{r}_A - m_B l\omega^2 + m_B r_A\omega^2$$

$$(M_A + M_B)\ddot{r}_A = (M_A + M_B)r_A\omega^2 - m_B l\omega^2$$

$$\boxed{\ddot{r}_A = r_A\omega^2 - \left(\frac{M_B}{M_A + M_B}\right)l\omega^2}$$

6) K&K 2.16

$$h = 6.6 \times 10^{-34} \frac{kg\, m^2}{s}$$

$$G = 6.7 \times 10^{-11} \frac{m^3}{kg\, s^2}$$

$$c = 3.0 \times 10^{8}\ m/s$$

a) $$L_p = \sqrt{\frac{hG}{c^3}} = \sqrt{\frac{(6.6\times10^{-34})(6.7\times10^{-11})}{(3.0\times10^{8})}} = 4.1\times10^{-35} \text{ meters}$$

b) $$m_p = \sqrt{\frac{hc}{G}} = \sqrt{\frac{(6.6\times10^{-34})(3.0\times10^{8})}{6.7\times10^{-11}}} = 5.4\times10^{-8}\ kg$$

c) $$t_p = \sqrt{\frac{hG}{c^5}} = \sqrt{\frac{(6.6\times10^{-34})(6.7\times10^{-11})}{(3\times10^{8})^5}} = 1.3\times10^{-43} \text{ sec}$$