Physics 3210, Spring 2019 Homework #1

Due *in class* Thursday January 17^{th}

1. Find the cosine and the sine of the angle between

$$\vec{A} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k} \qquad \text{and}$$
$$\vec{B} = -\hat{\imath} - \hat{\jmath} + 4\hat{k}$$

2. *K&K*[1] Problem 1.7. (See hint in back of book.)

3. Find a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$, from Problem 1 above.

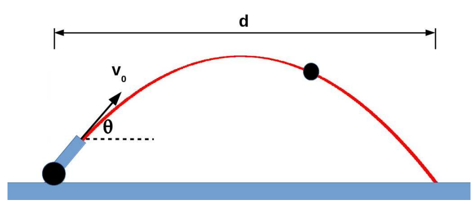
4. *K&K* Problem 1.10, but use the vector $\vec{A}$ from Problem 1 above.

5. A bomber plane flies horizontally over level terrain at 700 km/hr, at an altitude of 3.0 km, and drops a bomb. Neglect air resistance.

   (a) How far does the bomb travel horizontally between its release and hitting the ground?

   (b) If the bomber maintains its course, where is it when the bomb hits the ground?

6. A cannonball is fired with initial speed v_0 at an angle of θ with respect to the ground. Assume air resistance can be neglected.



   (a) Find an equation for the distance d traveled by the ball before it hits the ground, as a function of v_0, θ, and g.

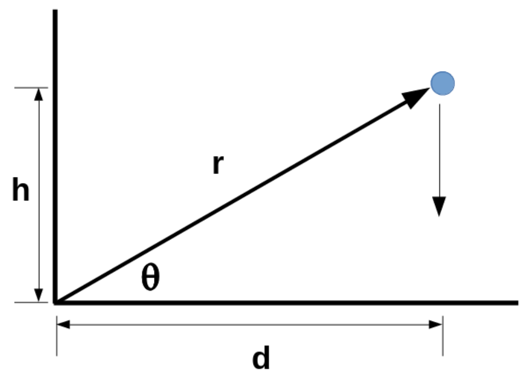
   (b) For which angle θ will the range d of the cannonball be maximized?

Problems continued on the next page.

[1] *K&K* $\equiv$ *Kleppner and Kolenkow, An Introduction to Mechanics*

7. *K&K* Problem 1.20. It will probably help to start by making sketches of speed versus time and position versus time for the car.

8. Consider uniform acceleration in polar coordinates. Suppose a ball is dropped at rest from a height h and horizontal distance d from the origin of coordinates. Find the radial speed dr/dt and angular speed $\omega = (d\theta/dt)$ just before the ball hits the ground.



9. *K&K* Problem 1.25.

10. *K&K* Problem 1.27.

Homework #1 Solutions

1) $\vec{A} = 3\hat{x} - \hat{y} + 2\hat{z}$

$\vec{B} = -\hat{x} - \hat{y} + 4\hat{z}$

Find cosine & sine of angle between $\vec{A}$ & $\vec{B}$

Compute magnitudes: $A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$

$B = \sqrt{\vec{B}\cdot\vec{B}} = \sqrt{(-1)^2 + (-1)^2 + 4^2} = \sqrt{18}$

From definition of "dot" product:

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(3)(-1) + (-1)(-1) + (2)(4)}{\sqrt{14}\sqrt{18}} = \boxed{0.37796}$$

& "cross" product:

$$|\sin\theta| = \frac{|\vec{A}\times\vec{B}|}{AB} \Rightarrow \vec{A}\times\vec{B} = [(-1)4 - (-1)2]\hat{x} + [2(-1) - 4\cdot 3]\hat{y} + [3(-1) - (-1)(-1)]\hat{z}$$

$$= -2\hat{x} - 14\hat{y} - 4\hat{z}$$

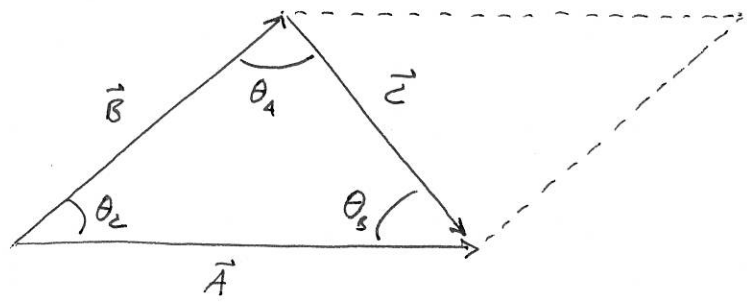
$$|\vec{A}\times\vec{B}| = \sqrt{(\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B})} = \sqrt{(-2)^2 + (-14)^2 + (-4)^2}$$

$$= \sqrt{216}$$

$$|\sin\theta| = \frac{\sqrt{216}}{\sqrt{14}\sqrt{18}} = 0.92582$$

$$\boxed{\sin\theta = \pm 0.92582}$$

2) K&K 1.7



$|\vec{A}\times\vec{B}|$ = (Area of Parallelogram) = 2 · (Area Triangle) = $AB \sin\theta_C$

$|\vec{B}\times\vec{C}|$ = " " $= BC \sin\theta_A$

$|\vec{C}\times\vec{A}|$ = " " $= CA \sin\theta_B$

so

$$AB \sin\theta_C = BC \sin\theta_A + CA \sin\theta_B$$

Divide thru by ABC to obtain

$$\boxed{\frac{\sin\theta_C}{C} = \frac{\sin\theta_A}{A} = \frac{\sin\theta_B}{B}}$$ "Law of sines"

3) FIND A <u>UNIT VECTOR</u> PERPENDICULAR TO BOTH $\vec{A} = 3\hat{x} + (-1)\hat{y} + 2\hat{z}$

$$\vec{B} = (-1)\hat{x} + (-\hat{y}) + 4\hat{z}$$

By Defn, THE <u>CROSS PRODUCT</u> of $\vec{A}$ & $\vec{B}$ WILL BE Perpendicular TO BOTH.

FROM PROBLEM 1, WE HAVE $\vec{A} \times \vec{B} = (-2)\hat{x} + (-14)\hat{y} + (-4)\hat{z}$

CREATE A <u>UNIT VECTOR</u> BY DIVIDING $\vec{A} \times \vec{B}$ BY ITS MAGNITUDE

AGAIN, FROM PROBLEM 1 $|\vec{A} \times \vec{B}| = \sqrt{216}$

So THE UNIT VECTOR WE WANT IS $\boxed{\hat{n} = \frac{1}{\sqrt{216}}\left(-2\hat{x} - 14\hat{y} - 4\hat{z}\right)}$

4) K & K 1.10, w/ $\vec{A} = 3\hat{x} + (-1)\hat{y} + 2\hat{z}$

$$A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$$

a) Let $\hat{B} = B_x \hat{x} + B_y \hat{y}$ (vector in xy plane)

$\hat{B} \perp \vec{A}$, so $\vec{A}\cdot\hat{B} = 3B_x + (-1)B_y = 0$

$$B_y = 3B_x$$

Now $\hat{B}$ is a unit vector, so $|\hat{B}| = \sqrt{\hat{B}\cdot\hat{B}} = \sqrt{B_x^2 + (3B_x)^2} = 1$

$$B_x = \sqrt{\tfrac{1}{10}}$$

$$B_y = 3\sqrt{\tfrac{1}{10}} = \sqrt{\tfrac{9}{10}}$$

$$\therefore \boxed{\hat{B} = \left(\sqrt{\tfrac{1}{10}}\,\hat{x} + \sqrt{\tfrac{9}{10}}\,\hat{y}\right)}$$

b) If $\hat{C} \perp \vec{A}$ and $\hat{C} \perp \hat{B}$, $\hat{C} = \dfrac{\vec{A}\times\hat{B}}{A}$

$$\vec{A}\times\hat{B} = \det\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & -1 & 2 \\ \sqrt{\frac{1}{10}} & \sqrt{\frac{9}{10}} & 0 \end{pmatrix} = \frac{1}{\sqrt{10}}\left(-6\hat{x} + 2\hat{y} + 10\hat{z}\right)$$

$$\boxed{\hat{C} = \frac{\vec{A}\times\hat{B}}{A} = \frac{1}{\sqrt{140}}\left(-6\hat{x} + 2\hat{y} + 10\hat{z}\right)}$$

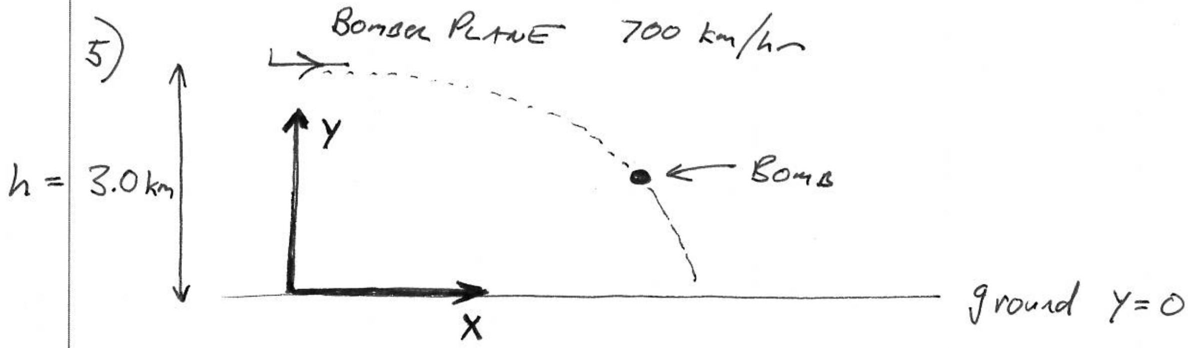
c) Now, $\hat{B}\times\hat{C}$ is perpendicular to plane containing $\hat{B}$ & $\hat{C}$

$$\hat{B}\times\hat{C} = \frac{1}{\sqrt{10}}\cdot\frac{1}{\sqrt{140}}\cdot\det\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 3 & 0 \\ -6 & 2 & 10 \end{pmatrix}$$

$$= \frac{1}{\sqrt{1400}}\left(30\hat{x} + (-10)\hat{y} + 20\hat{z}\right)$$

Clearly $\hat{B}\times\hat{C}$ is proportional to $\vec{A}$, so <u>$\vec{A}$ is perpendicular to plane</u>

5)



a) $X = X_0 + V_{0x} t$ ← NO ACCELERATION IN X

$y = y_0 + V_{0y} t + \frac{1}{2} a t^2$ ← constant acceleration at 'g' in Y, $V_{0y} = 0$

$= h - \frac{1}{2} g t^2$

$Y_{ground} = h - \frac{1}{2} g t^2 = 0$

$h = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 3000m}{9.81 m/s^2}} = 24.731 \text{ sec}$

DISTANCE TRAVELLED BY BOMB $d = X - X_0 = V_{0x} t$

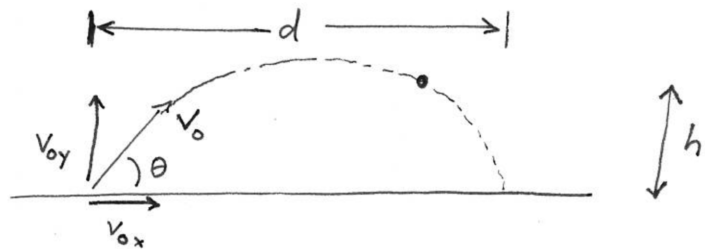
$= \left(700 \frac{km}{hr}\right)\left(\frac{1000 m}{km}\right) \cdot \left(\frac{1 hr}{3600 s}\right) \cdot 24.731 s$

$= 4,809 m$

$\boxed{d = 4.809 km}$

b) IF THE BOMBER MAINTAINS COURSE, IT IS <u>DIRECTLY OVER THE BOMB</u> WHEN IT HITS THE GROUND.

6)



a) $$d = x - x_0 = V_{ox} t = V_0 \cos\theta \, t$$

$$h = y - y_0 = V_{oy} t - \tfrac{1}{2} g t^2 = 0$$ (WHEN BALL HITS GROUND)

$$V_0 \sin\theta = \tfrac{1}{2} g t$$

$$t = \frac{2V_0}{g} \sin\theta$$

AND $$\boxed{d = \frac{2V_0^2}{g} \cos\theta \sin\theta}$$

b) Find Angle θ giving max d by setting

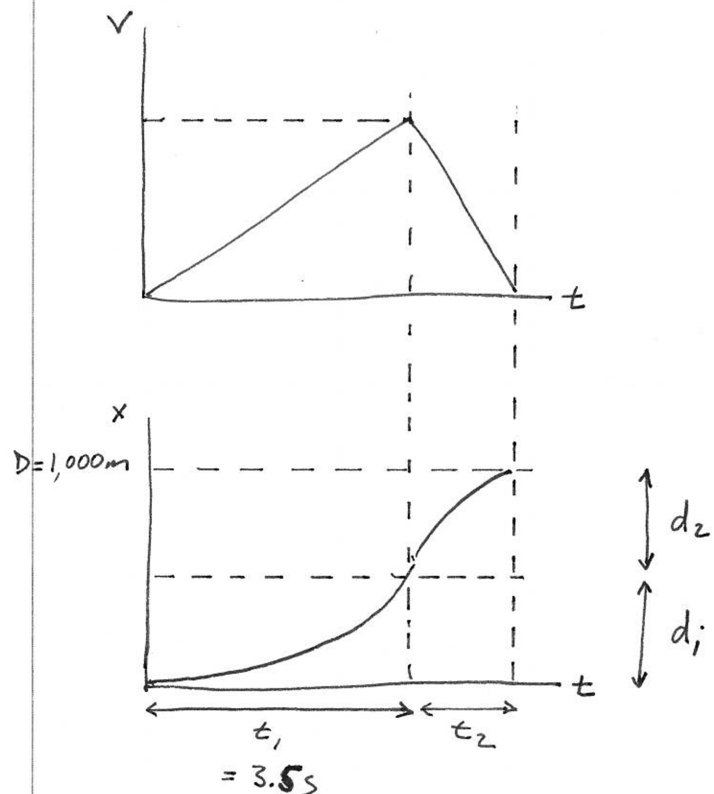
$$\frac{d}{d\theta} d = \frac{2V_0^2}{g} \left[-\sin\theta \sin\theta + \cos\theta \cos\theta \right] = 0$$

$$\cos^2\theta = \sin^2\theta$$

$$\boxed{\theta = 45^\circ}$$

7) K & K Problem 1.20

Sketch speed & position:



Car accelerates at

$$a_1 = \frac{100 \frac{km}{hr}}{3.5\ s} \times \left(\frac{1\ hr}{3600\ s}\right) \times \left(\frac{1000\ m}{km}\right)$$

$$= 7.9365\ m/s^2$$

$$a_2 = 0.7 \times 9.81\ \frac{m}{s^2} = 6.867\ m/s^2$$

Now, $D = d_1 + d_2 = \frac{1}{2} a_1 t_1^2 + V_{MAX} t_2 - \frac{1}{2} a_2 t_2^2$

Also $\quad V_{MAX} = a_2 t_2\ ,\quad a_1 t_1 = a_2 t_2\ ,\quad t_1 = \frac{a_2}{a_1} t_2$

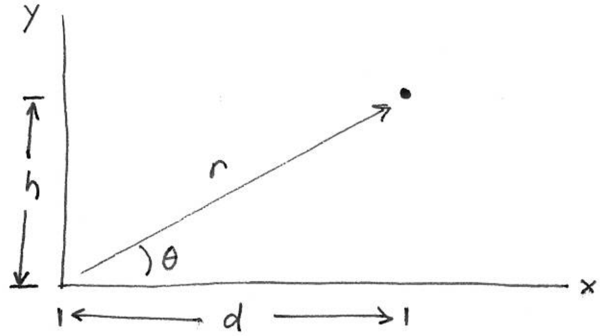
So $\quad D = \frac{1}{2} a_1 \left[\frac{a_2}{a_1} t_2\right]^2 + a_2 t_2^2 - \frac{1}{2} a_2 t_2^2$

$\Big\downarrow$ arithmetic

$$T = t_1 + t_2 = \sqrt{\frac{2D(a_1 + a_2)}{a_1 a_2}} = \sqrt{\frac{2 \cdot 1000\ m \cdot (7.9365 + 6.867)\ m/s^2}{(7.9365 \cdot 6.867)\ (m/s^2)^2}}$$

$$\boxed{T = 23.31\ sec}$$

8)



$$r = (x^2+y^2)^{1/2} \qquad x = x_0 = d \quad \text{(NO ACCEL IN } x\text{)}$$

$$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2$$

$$= h - \tfrac{1}{2}gt^2$$

$$\therefore \quad r = \left[d^2 + (h - \tfrac{1}{2}gt^2)^2\right]^{1/2}$$

and, WHEN BALL HITS ground

$$y = 0$$

$$t = \sqrt{\frac{2h}{g}}$$

$$\frac{dr}{dt} = \frac{-(h - \frac{1}{2}gt^2)gt}{\left[d^2 + (h-\frac{1}{2}gt^2)^2\right]^{1/2}}$$

$$\left.\frac{dr}{dt}\right|_{t=\sqrt{\frac{2h}{g}}} = \frac{-\left(h - \frac{1}{2}g\cdot\left(\frac{2h}{g}\right)\right)g\sqrt{\frac{2h}{g}}}{\left[d^2 + \left(h - \frac{1}{2}g\cdot\frac{2h}{g}\right)^2\right]^{1/2}} = 0$$

$$\boxed{\left.\frac{dr}{dt}\right|_{t=\sqrt{\frac{2h}{g}}} = 0}$$

$$\theta = \text{Arctan}\left(\frac{y}{x}\right) = \text{ARCTAN}\left(\frac{h - \frac{1}{2}gt^2}{d}\right)$$

$$\omega = \frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{h - \frac{1}{2}gt^2}{d}\right)^2}\cdot\left(\frac{-gt}{d}\right)$$

$$\boxed{\omega\left(t = \sqrt{\frac{2h}{g}}\right) = -\frac{\sqrt{2gh}}{d}}$$

9) K & K 1.25

$$\theta = \tfrac{1}{2}\alpha t^2 \qquad r = \frac{\theta}{\pi} = \frac{\alpha t^2}{2\pi}$$

$$\dot\theta = \alpha t \qquad \dot r = \frac{\alpha t}{\pi}$$

$$\ddot\theta = \alpha \qquad \ddot r = \alpha/\pi$$

$$\vec V = \dot r\hat r + r\dot\theta\hat\theta = \frac{\alpha t}{\pi}\hat r + \frac{\alpha^2 t^3}{2\pi}\hat\theta$$

$$\vec a = \left[\ddot r - r\dot\theta^2\right]\hat r + \left[r\ddot\theta + 2\dot r\dot\theta\right]\hat\theta = \left[\frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi}\right]\hat r + \left[\frac{5\alpha^2 t^2}{2\pi}\right]\hat\theta$$

a) See attached sketch.

b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{\sqrt{2}}$; $t^2 = \frac{\sqrt{2}}{\alpha}$, $t^4 = \frac{2}{\alpha^2}$

Radial acceleration (see above) $= \frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi} = \frac{\alpha}{\pi} - \frac{\alpha^3}{2\pi}\cdot\frac{2}{\alpha^2} = 0$

c) Radial & tangential accelerations have equal magnitudes when either

$$\frac{\alpha}{\pi} - \frac{\alpha^3 t^4}{2\pi} = \frac{5\alpha^2 t^2}{2\pi} \quad \text{or} \quad \frac{\alpha^3 t^4}{2\pi} - \frac{\alpha}{\pi} = \frac{5\alpha^2 t^2}{2\pi}$$

Now $\theta = \frac{1}{2}\alpha t^2$; $t^2 = 2\theta/\alpha$

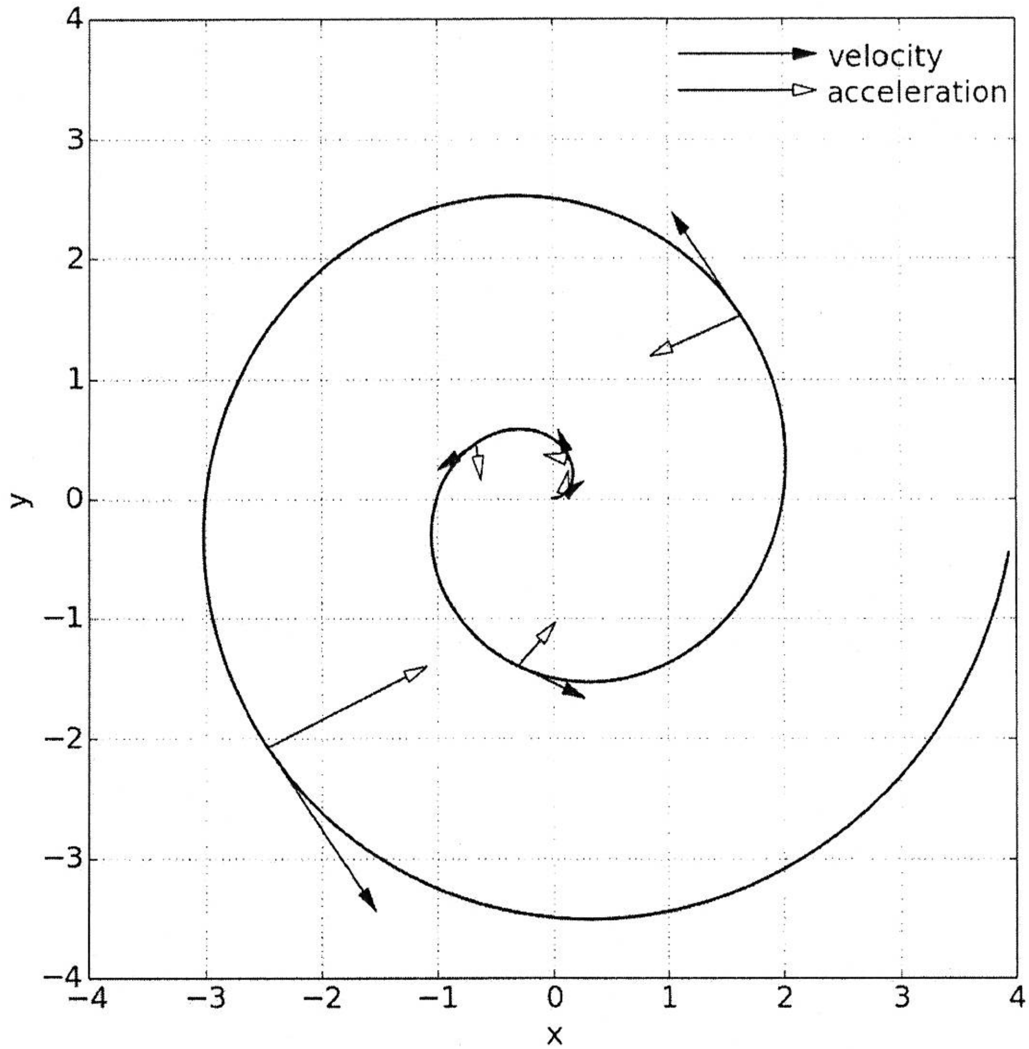
so

$$\frac{\alpha}{\pi}\left(1 - 2\theta^2\right) = \frac{\alpha}{\pi}\cdot 5\theta \qquad \frac{\alpha}{\pi}\left(2\theta^2 - 1\right) = \frac{\alpha}{\pi}\cdot 5\theta$$

$$\theta = \frac{-5 \pm \sqrt{25+8}}{4} \qquad \theta = \frac{5 \pm \sqrt{25+8}}{4}$$

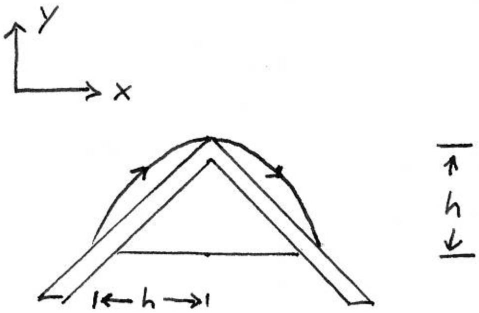
$\theta > 0$, so only need positive root

$$\boxed{\theta = 0.186 \text{ rad} = 10.66^\circ \qquad \theta = 2.686 \text{ rad} = 153.90^\circ}$$



velocity
acceleration
4
3
2
1
0
−1
−2
−3
−4
y
−4
−3
−2
−1
0
1
2
3
4
x

10) K & K Problem 1.27



Symmetric peaked roof, subtending right angle.

Standing a distance h below the peak, with what initial speed must ball be thrown so that it just clears the peak and hits the other side of the roof at the same height?

Write down kinematic equations in Cartesian coordinates for uniform acceleration $\vec{a} = -g\hat{y}$:

$$X = X_0 + V_{0x} t$$

$$V_x = V_{0x}$$

$$Y = Y_0 + V_{0y} t - \frac{1}{2} g t^2$$

$$V_y = V_{0y} - gt$$

Since roof is left-right symmetric, first half of parabola is mirror image of 2^{ND}.

Consider only 1^{ST} half...

(i) $X - X_0 = h = V_{0x} t$

(ii) $V_x = V_{0x}$

(iii) $Y - Y_0 = h = V_{0y} t - \frac{1}{2} g t^2$

(iv) $V_y = 0 = V_{0y} - gt$

3 Eqns in 3 unknowns V_{0x}, V_{0y}, t

$\Rightarrow$ Eliminate t with (iv) $t = V_{0y}/g$

$\Rightarrow$ Plug into (i) $h = \frac{V_{0x} V_{0y}}{g}$

$\Rightarrow$ Plug into (iii) $h = \frac{V_{0y}^2}{g} - \frac{1}{2}\frac{V_{0y}^2}{g} = \frac{1}{2}\frac{V_{0y}^2}{g}$; $\underline{V_{0y} = \sqrt{2gh}}$

$$V_{0x} = \frac{gh}{V_{0y}} = \sqrt{\frac{gh}{2}}$$

Initial speed $V_0 = \sqrt{V_{0x}^2 + V_{0y}^2} = \sqrt{\frac{gh}{2} + 2gh}$

$$\boxed{V_0 = \sqrt{\frac{5}{2} gh}}$$