Physics 3210, Spring 2018

Final Exam

Name:

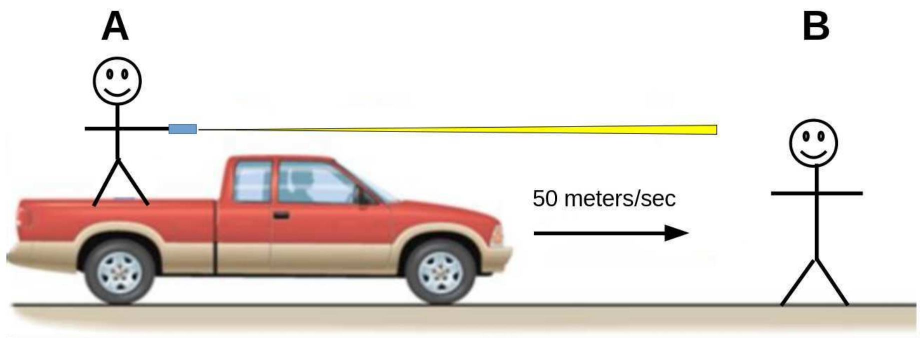
Signature:

UID:

Please read the following before continuing:

- Show all work in answering the following questions. Partial credit may be given for problems involving calculations.
- Be sure that your final answer is clearly indicated, for example by drawing a box around it.
- Be sure that your cellphone is turned off.
- Your signature above indicates that you have neither given nor received unauthorized assistance on any part of this exam.
- Thanks, and good luck!

1. (6 pts) Alice stands in the back of a pickup truck which is moving towards Bob at 50 meters/sec. She shines a flashlight towards Bob. Indicate if the statements below are true or false. $c = 299,792,458$ m/s is the speed of light.



(a) (1) Alice measures the beam traveling at $(c - 50)$ m/sec, since she is "catching up" with the light beam. **T** **F**.

(b) (1) Bob alone will measure the flashlight beam travelling at c, since his reference frame is at rest and hence "privileged". **T** **F**.

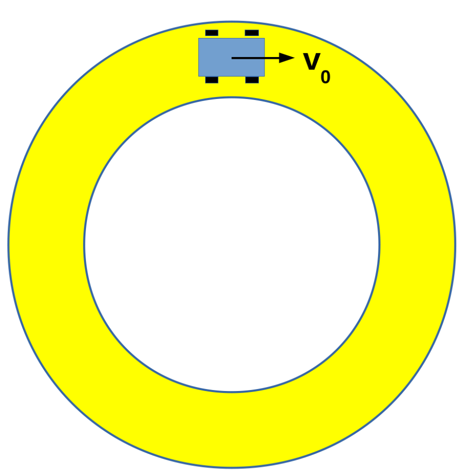
(c) (1) Alice and Bob measure the light beam travelling at the same speed c. **T** **F**.

(d) (1) Alice alone will measure the flashlight beam travelling at c, since she is carrying the source of light and hence her frame is "privileged". **T** **F**.

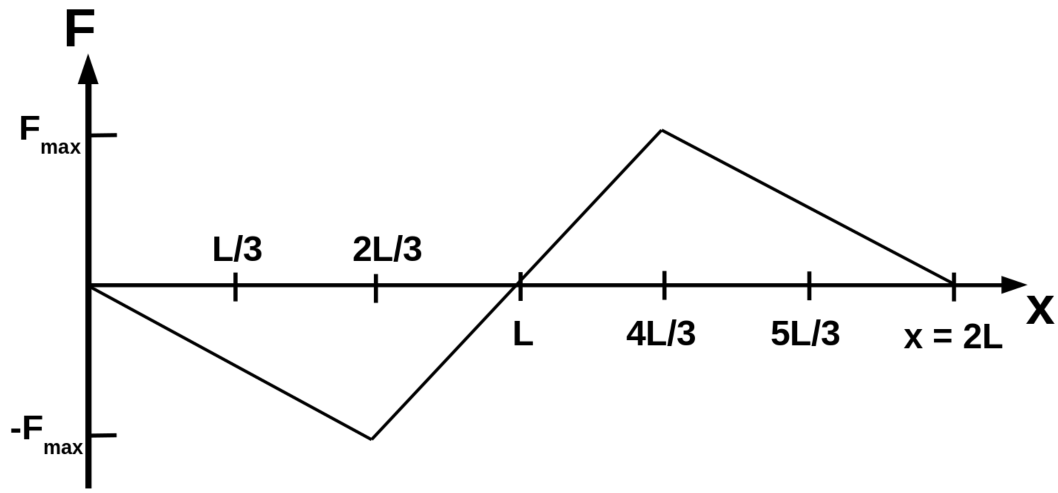
(e) (1) Bob measures the beam traveling at $(c + 50)$ m/sec. **T** **F**.

(f) (1) The answers to the above could change if the pickup truck were moving closer to the speed of light. **T** **F**.

2. (3 pts) A car is driving on a circular track as shown in the figure below. At the instant shown, the driver steps on the gas so as to increase the car's tangential speed. On the figure, sketch an arrow representing the direction of the net frictional force being applied to the track by the car's wheels.



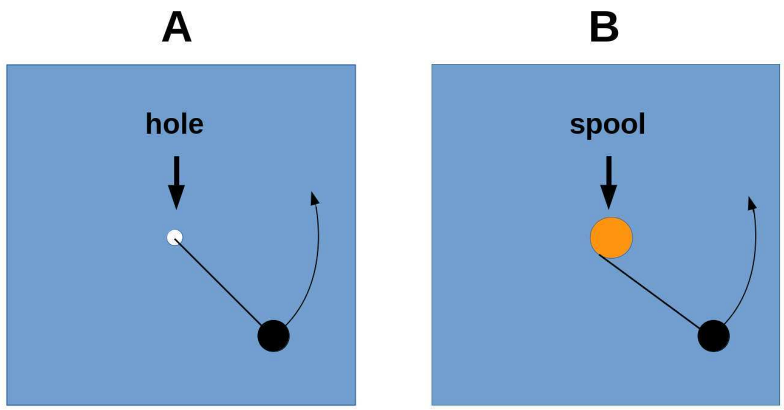
3. (6 pts) A variable force is applied to a particle of mass m, which is initially at rest, as illustrated in the graph below.



(a) (3) What is the velocity (magnitude and direction) of the mass at $x = L$ and $x = 2L$?

(b) (3) Give the x-coordinate of all points at which the *acceleration* of the mass is equal to zero.

4. (10 pts) Two identical hockey pucks are sliding on frictionless tables. They are viewed from above in Figures A and B below. Both pucks are attached to massless strings. In the Figure A, the string passes through a hole and someone applies a downward tension force to the string as the puck “orbits” the hole. In Figure B, the string winds around a vertical pole as the puck orbits. In both cases, the initial angular speed is ω_0 and the radius of the orbit is r_0. Answer the questions below.



(a) (3) In Figure A, is angular momentum conserved? Kinetic energy? Explain.

(b) (3) In Figure B, is angular momentum conserved? Kinetic energy? Explain.

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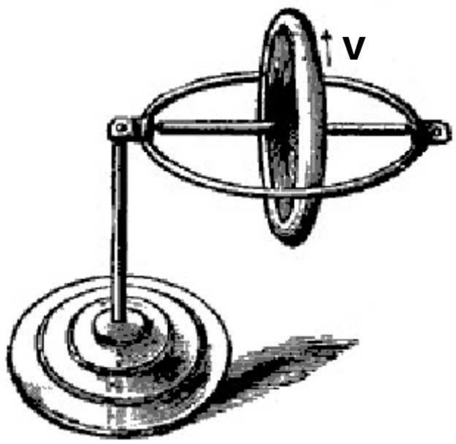
(c) (4) Which puck will have a higher speed when the radius is $r_0/2$? By what factor?

5. (8 pts) Consider the gyroscope shown in the figure. A wheel of mass M rotates upon its axis. The edge of the wheel has tangential speed v in the direction shown. The wheel is a distance D from its point of support.



Indicate the direction of the following from your viewing perspective as "left", "right", "into the page", or "out of the page":

(a) (2) Angular momentum vector ______________________________

(b) (2) Torque due to gravity ______________________________

(c) (2) Precession of gyroscope ______________________________

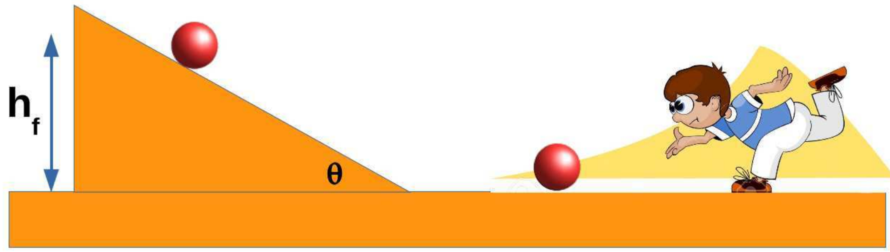
Also please provide a short answer to the following:

(d) (2) Which of the above would be **unchanged** if you solved this problem using the "left-hand rule"? Explain.

6. (3 pts) An object on the end of a string is executing circular motion, with a tangential speed v. If the length of the string is tripled and the tangential speed remains unchanged, what would happen to the tension in the string?

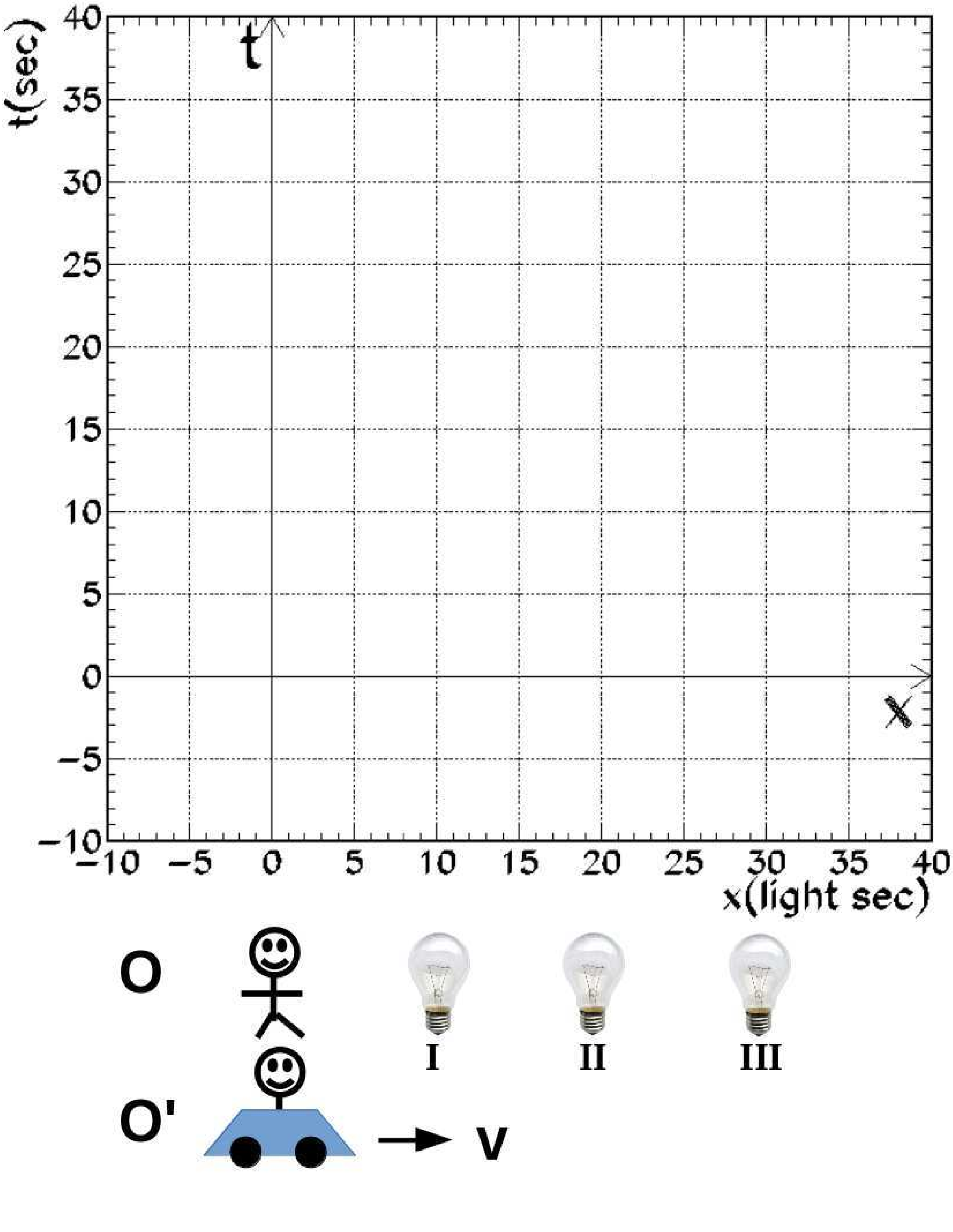
7. (3 pts) If the Earth's polar ice caps continue to melt, it is likely that the melted water will be distributed closer to the equator. Due to this effect, would the length of Earth's day tend to (a) increase (b) decrease or (c) stay the same? Explain.

8. (6 pts) A bowler throws a spherical ball of mass m and radius R with initial speed v. It rolls without slipping down the lane and then up a ramp making an angle θ with respect to horizontal.



Calculate the final height h_f to which the ball rolls up the ramp.

9. (12 pts) At $t = 0$ Sandy (frame $\mathcal{O}$) and Pat (frame $\mathcal{O}'$) are together at the origins of their respective coordinates. Pat is sitting in a race car, which is moving at speed $v = c/2$ relative to Sandy, past street lamps I, II and III placed at 10 light-second intervals in frame $\mathcal{O}$.



Please follow the instructions on the following page.

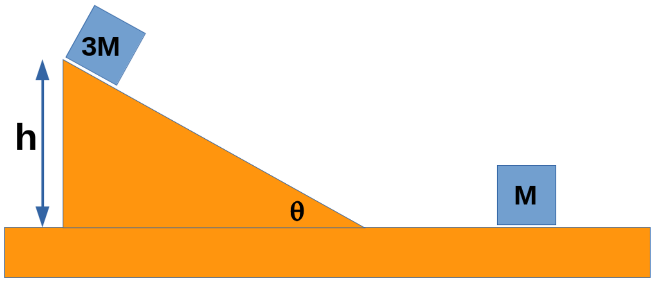
(a) (3) On the space-time diagram, draw **and label** the world line of Pat. Call this the t' axis. Also draw and label the x' axis.

(b) (3) Draw the world lines of street lamps I, II and III. These lamps turn on at 10, 15, and 20 seconds respectively in frame $\mathcal{O}$. Label the event of each lamp turning on as A, B, and C in the diagram.

(c) (3) In frame $\mathcal{O}'$, which occurs first, event A or B? Or do they occur at the same time? Explain.

(d) (3) What is the spacing of the streetlamps in frame $\mathcal{O}'$?

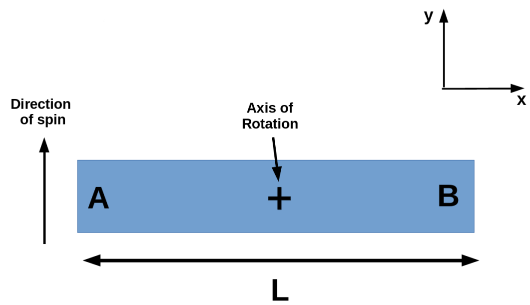
10. (10 pts) A block of mass $3M$ is released from rest at a height h above the bottom of a frictionless ramp, which makes an angle $\theta = 30°$ with respect to horizontal. See the figure below. It slides down the ramp then across a frictionless table, and collides elastically with a second block of mass M which is initially at rest.



(a) (4) If the speed of the mass $3M$ is 1.0 m/s when it reaches the bottom of the ramp, what is the height h (in centimeters) from which it was released?

(b) (6) What are the final velocities (magnitude and direction) of the two blocks after the elastic collision?

11. (12 pts) Annie (A) and Bob (B) stand near either end of a freely rotating beam of length $L = 4.0$ m. Annie's mass is 75 kg and Bob's is 100 kg, and the mass of the beam is 200 kg. The beam spins clockwise at $(1/\pi)$ revolutions per second. The $+x$ and $+y$-directions in the beam's rotating frame are as indicated. The $+z$-direction is out of the page.



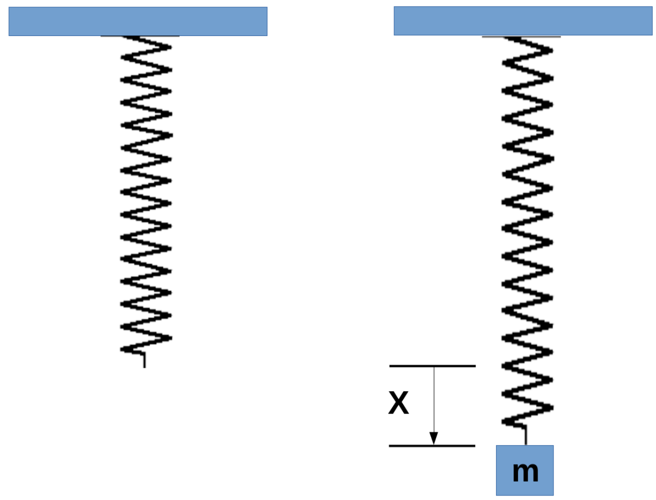
(a) (3) What are the magnitude and direction of the fictitious "centrifugal force" on Bob? What is the magnitude and direction of the non-fictitious centripetal force on his feet?

(b) (3) If Bob walks towards the center of the beam and Annie stays where she is, what is the final rate of spin of the beam? (Assume Bob stands on a point at the beam's axis of rotation.)

(c) (3) What is the *direction* of the Coriolis force on Bob as he walks?

(d) (3) What is the *direction* of the azimuthal force on Annie as Bob walks?

12. (9 pts) A massless spring (spring constant k) hangs vertically from the ceiling (left). A mass m is then attached, and the spring is stretched relative to its initial length. Call x_0 the distance the mass-supporting spring is stretched relative to its unstretched position, when it is not oscillating (right).



(a) (3) Find x_0 in terms of k, m and g.

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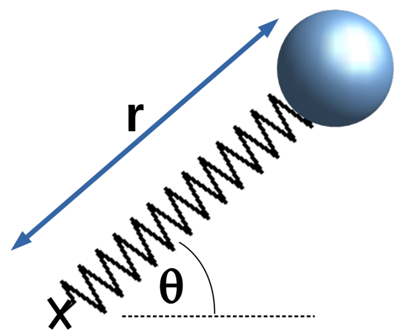
(b) (3) Now, the mass is set into vertical oscillations. Find the equation of motion of the mass in the coordinate x, relative to the position of the end of the unstretched spring.

(c) (3) Show that, with the change of variables $y \equiv (x - x_0)$, the system obeys the simple harmonic oscillator equation. What is the period of oscillation of the system?

13. (12 pts) A 3-dimensional harmonic oscillator consists of a mass m on one end of a "spring", the other end of which is (somehow) fixed to a point in space. For the purposes of this problem, assume the mass to be pointlike. The force on the particle is given by $\vec{F}(r) = -kr\hat{r}$, *i.e.* at the equilibrium position the spring has zero length.



(a) (2) Explain why we expect the motion of the oscillator to be in a plane, that is, the two coordinates r and θ shown in the figure are sufficient to describe its motion.

(b) (2) Write down expressions for the kinetic and potential energy of this system in polar coordinates.

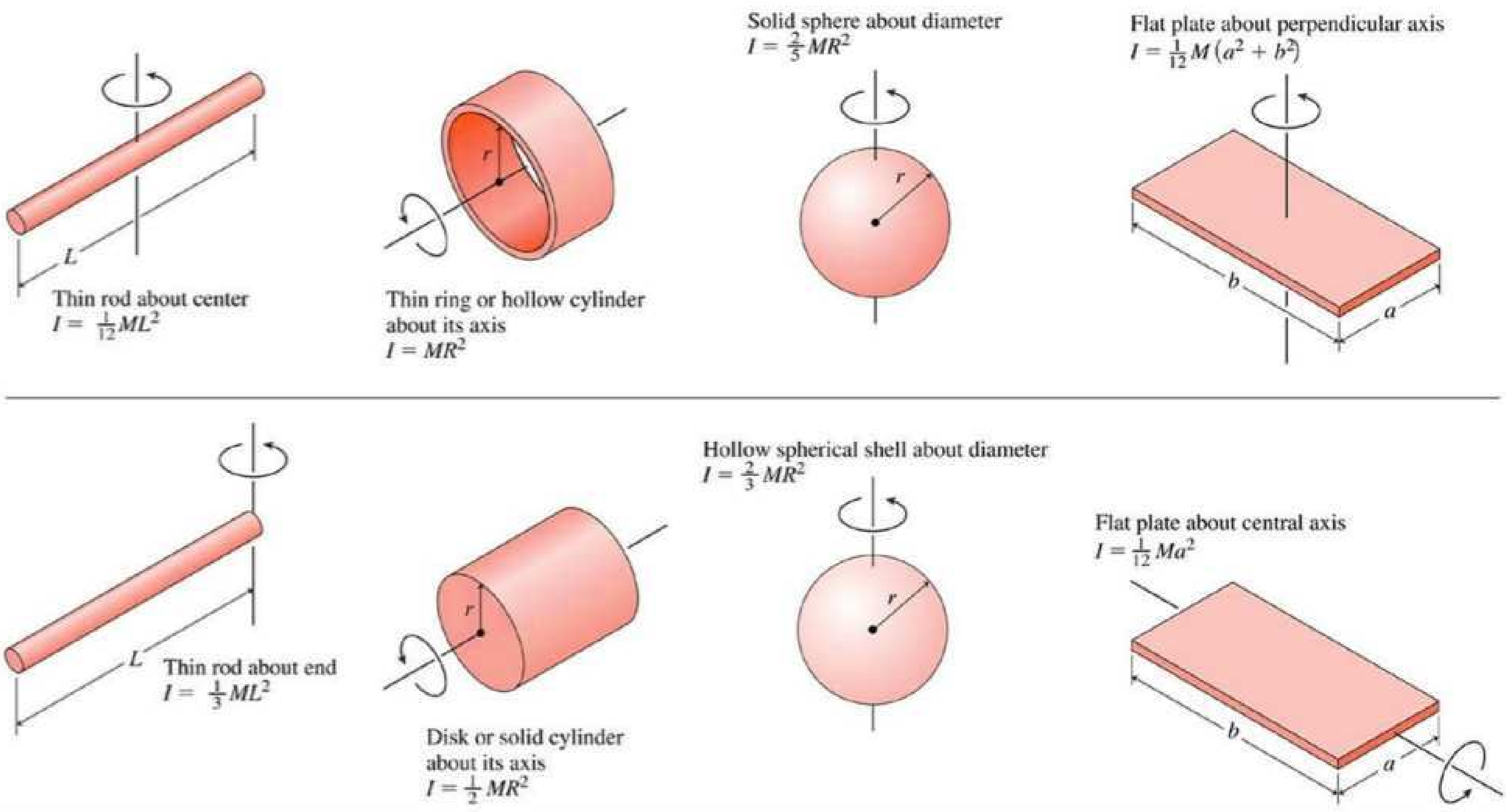
(c) (2) Use the Lagrangian method to confirm that the angular momentum

$$L = mr^2\dot{\theta} = mr^2\omega$$

is a constant of the motion.

(d) (2) Write down (or derive) an expression for the effective potential U_{eff} of this central force problem.

(e) (4) Find the radius of a circular orbit as a function of L, m, and k. What angular speed must the mass have in order to have a circular orbit?


Thin rod about center
$I = \frac{1}{12}ML^2$
L
Thin ring or hollow cylinder
about its axis
$I = MR^2$
r
Solid sphere about diameter
$I = \frac{2}{5}MR^2$
r
Flat plate about perpendicular axis
$I = \frac{1}{12}M(a^2 + b^2)$
b
a
Thin rod about end
$I = \frac{1}{3}ML^2$
L
Disk or solid cylinder
about its axis
$I = \frac{1}{2}MR^2$
r
Hollow spherical shell about diameter
$I = \frac{2}{3}MR^2$
r
Flat plate about central axis
$I = \frac{1}{12}Ma^2$
b
a