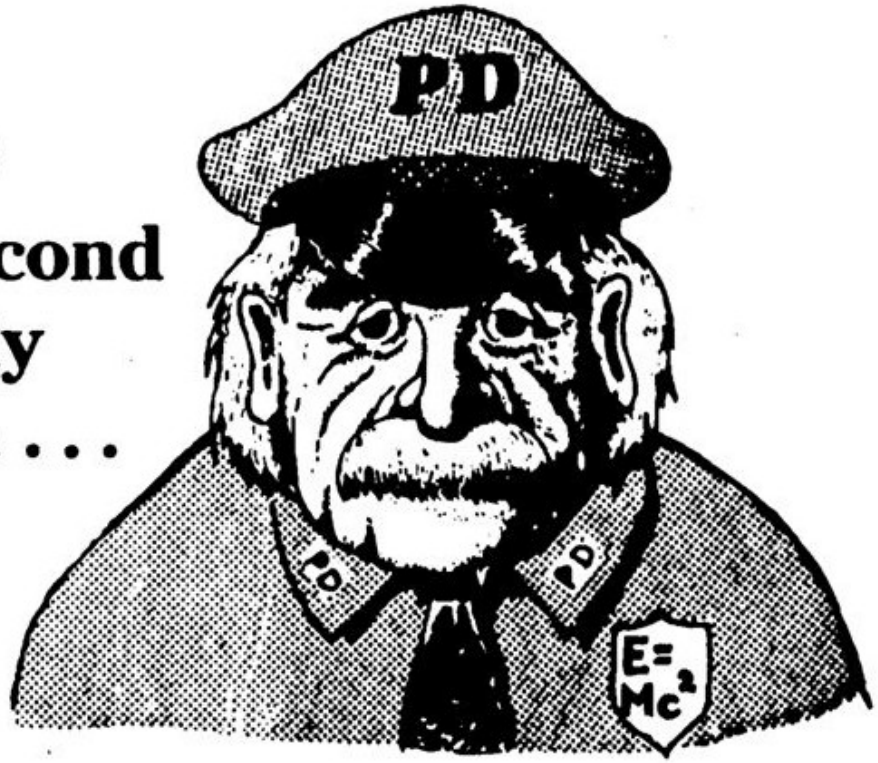
186,000
miles per second
is not only
a good idea . . .



IT'S THE LAW!

1

is not only
a good idea . . .

IT'S THE LAW!

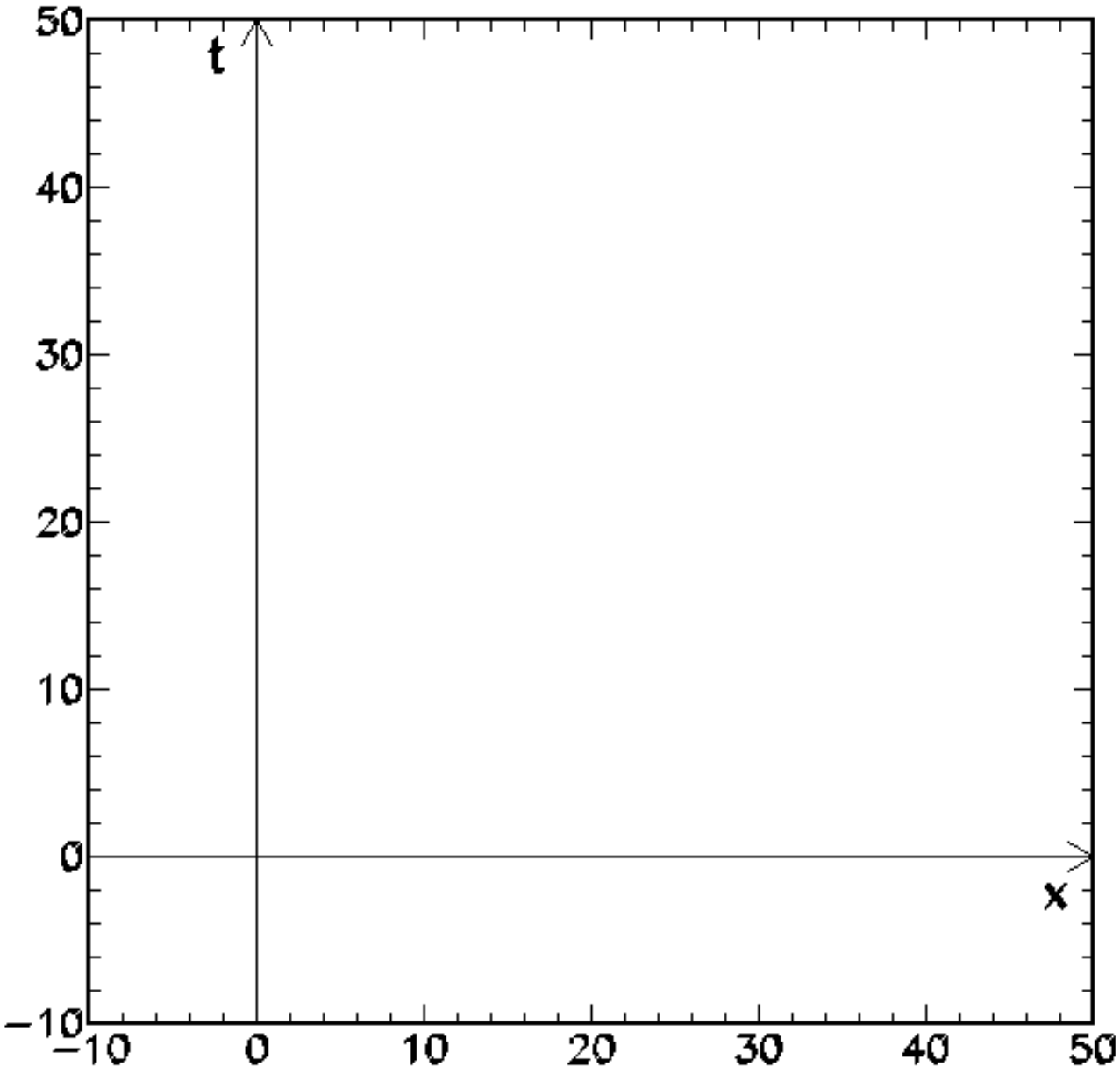
3. A lightweight pole 20 m long lies on the ground next to a barn 15 m long. An Olympic athlete picks up the pole, carries it far away, and runs with it towards the end of the barn at a speed of 0.8 c. Her friend remains at rest, standing by the barn door.

   (a) How long does the friend measure the pole to be, as it approaches the barn?

   (b) The barn door is initially open, and immediately after the runner and pole are entirely inside the barn, the friend shuts the door. How long after the door is shut does the front of the pole hit the other end of the barn, as measured by the friend?

   (c) In the reference frame of the runner, what is the length of the barn and the pole?

   (d) Does the runner believe that the pole is entirely inside the barn when it hits the end of the barn? Can you explain why, and the apparent contradiction between what is seen by the runner and what is seen by her friend?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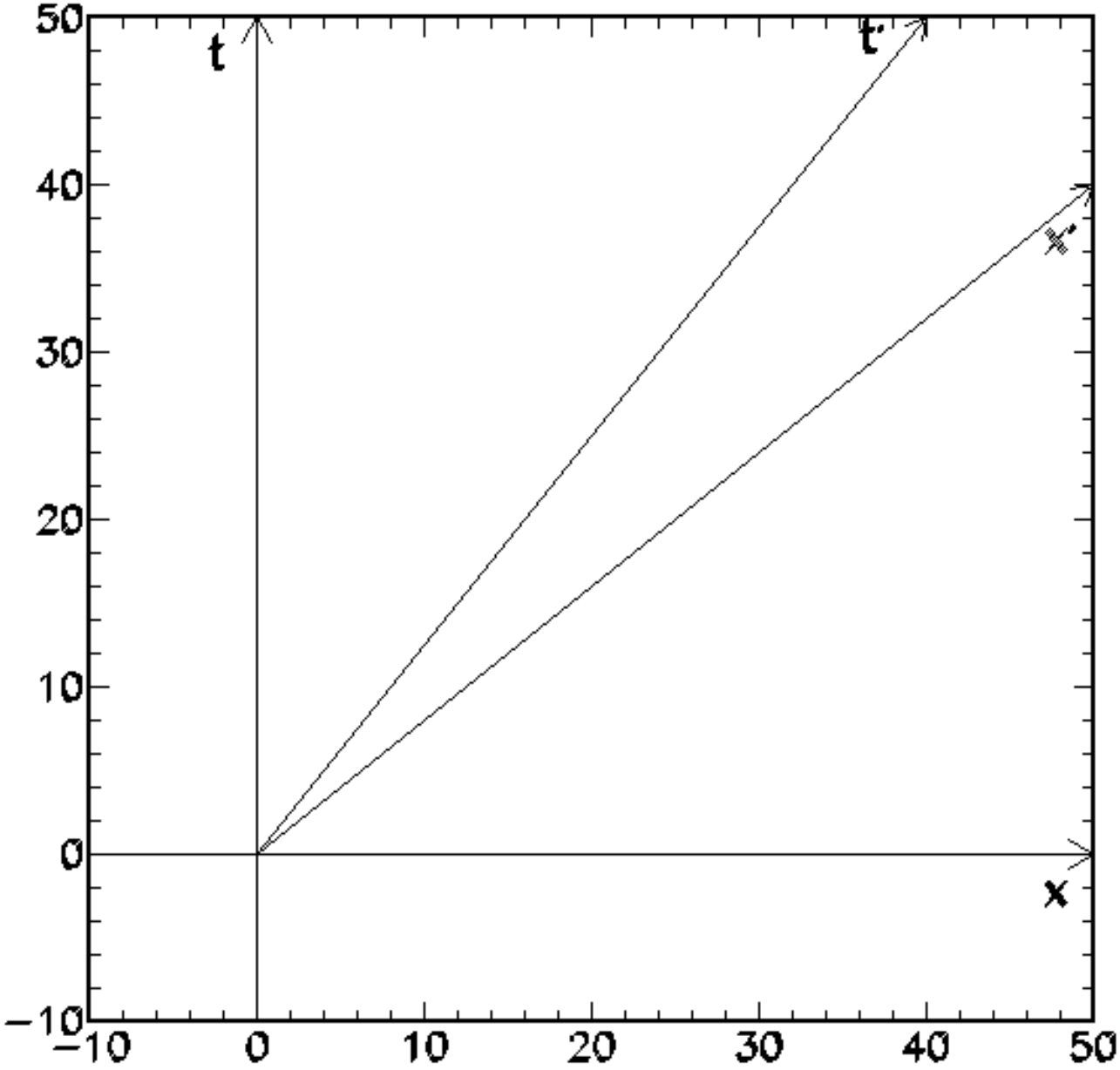
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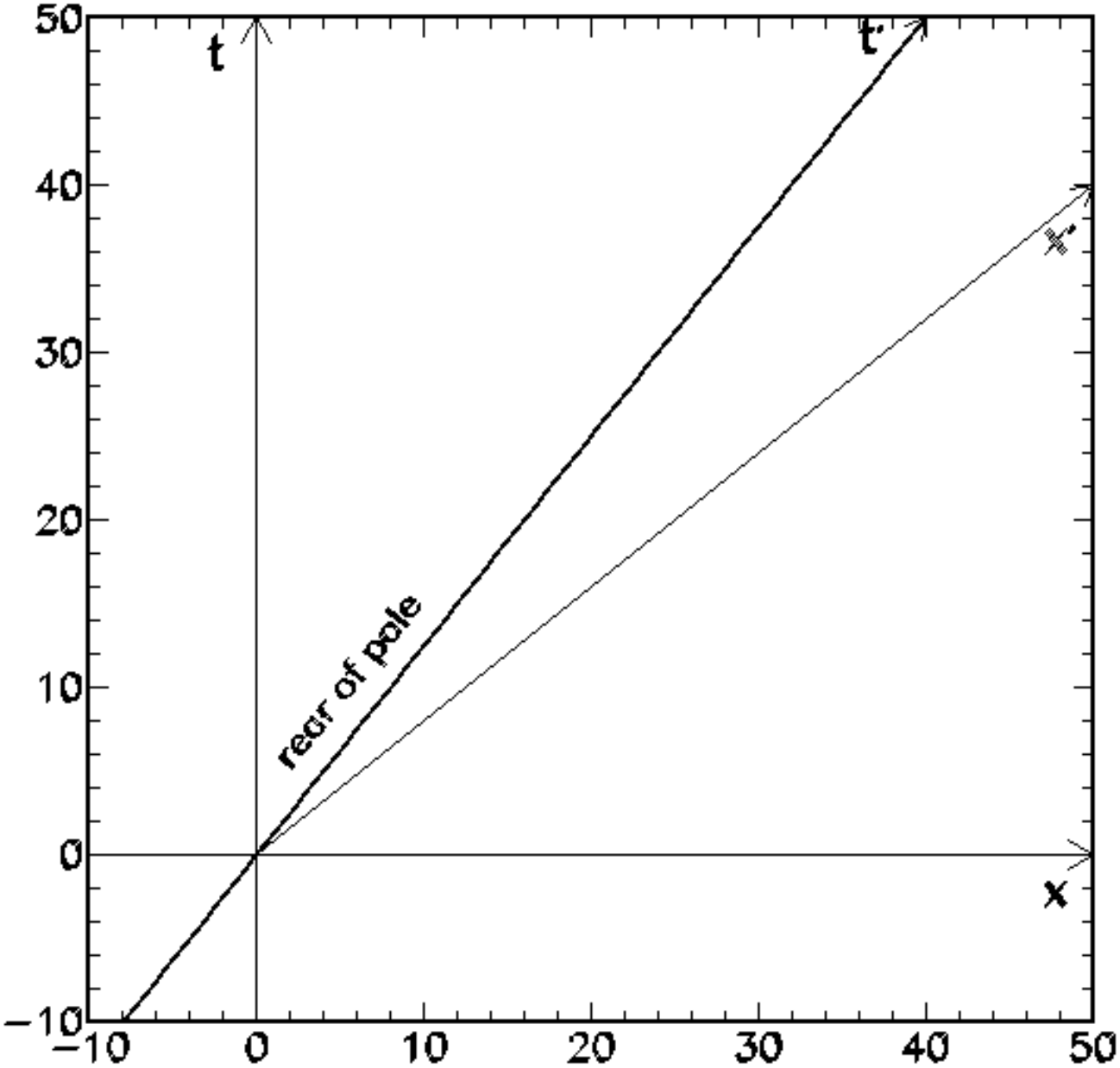
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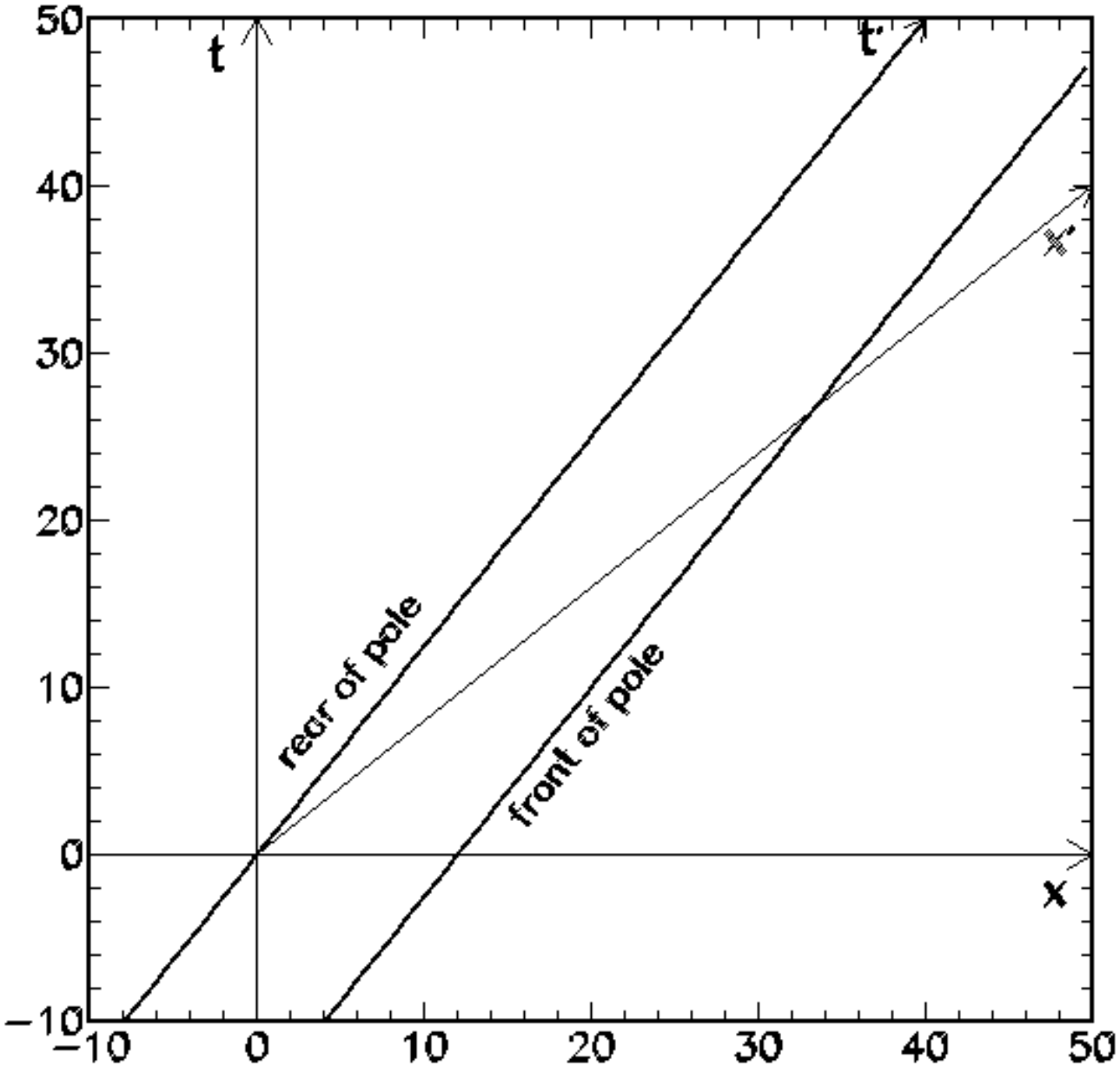
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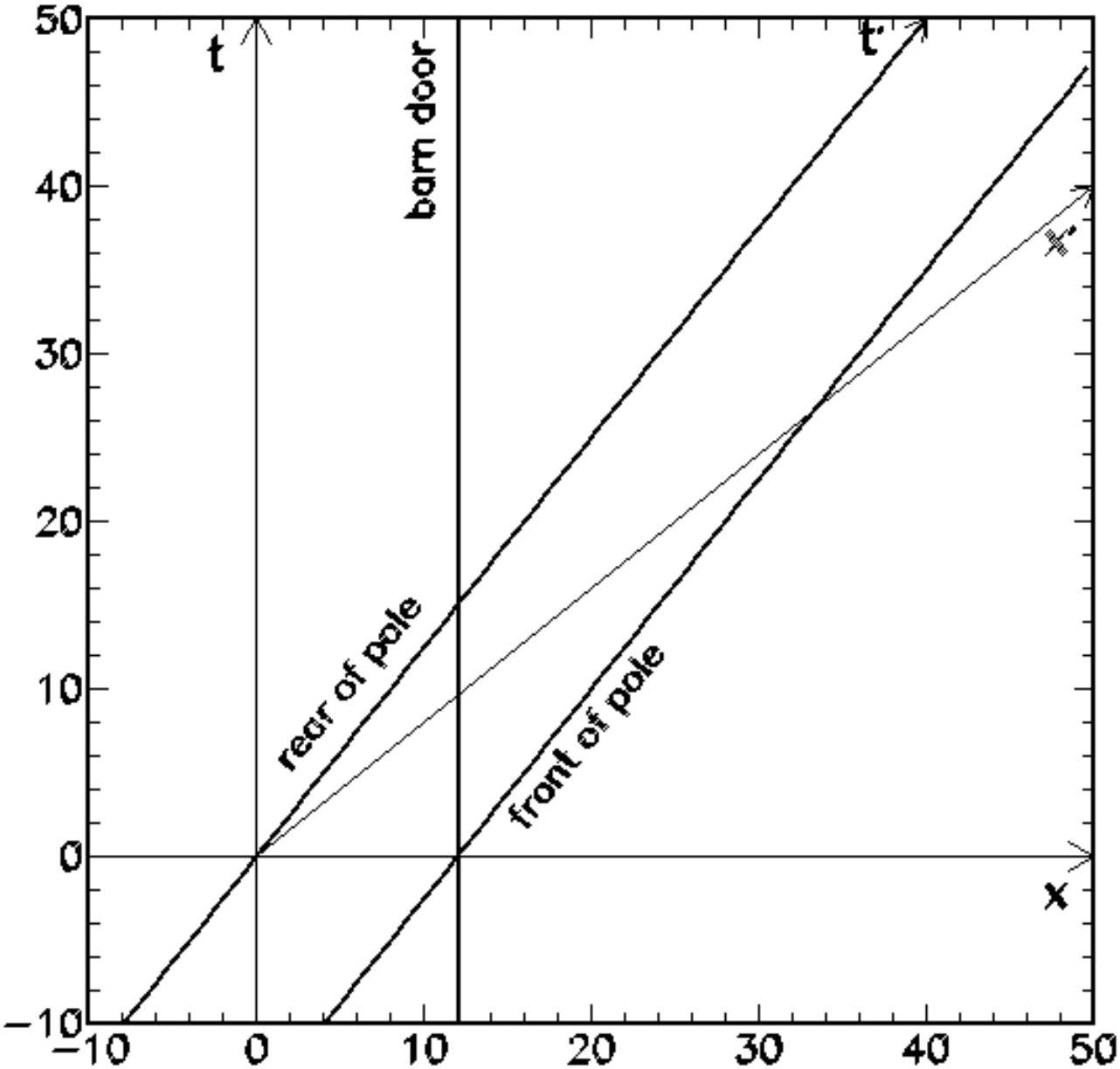
t
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barn door
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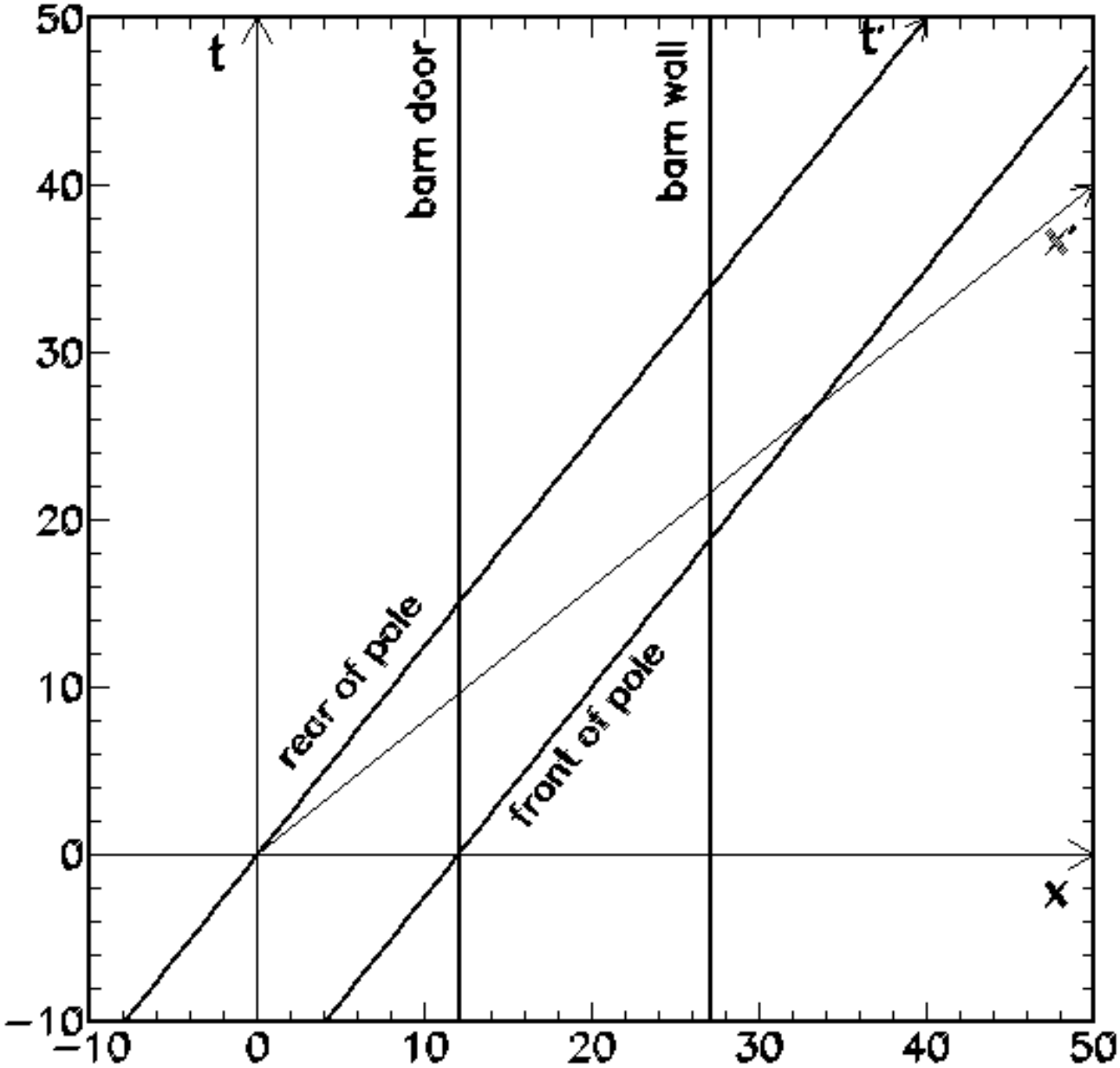
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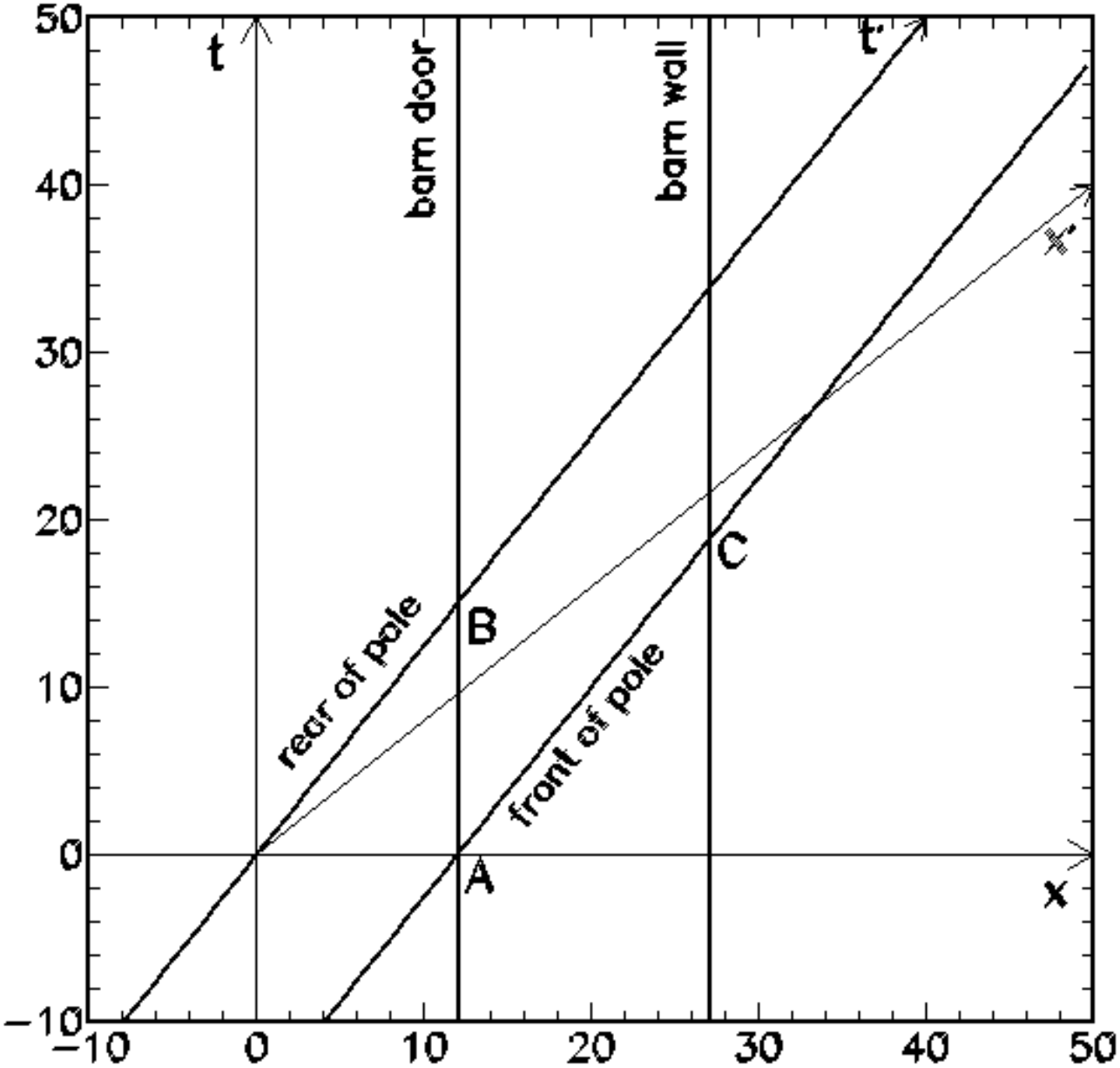
t
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Course Review

Physics 3210
Spring Semester 2019

- Today: Course Review
- Tomorrow: Final discussion session (space-time diagrams)
- By Tuesday 23rd: Take practice final!
  - We'll review answers in class
- Thursday 25th: Final Exam

Final Exam Details

- When: Thursday April 25th, 1:00-3:00 PM.
- Where: JFB B1… here!
- Allowed materials:
  - Equation sheet
  - Pen/pencil(s)
  - Hand held calculator. Not the one on your phone.
  - Straightedge to make neat diagrams

More Details

- Coverage: All Units
- Types of problems: Short answer *and* workout.
- Study recommendations:
  - Review homework
  - Review prior exams and practice exams
  - Exercises from lectures and discussion (online)
  - *Alles leben ist problemlösen…*

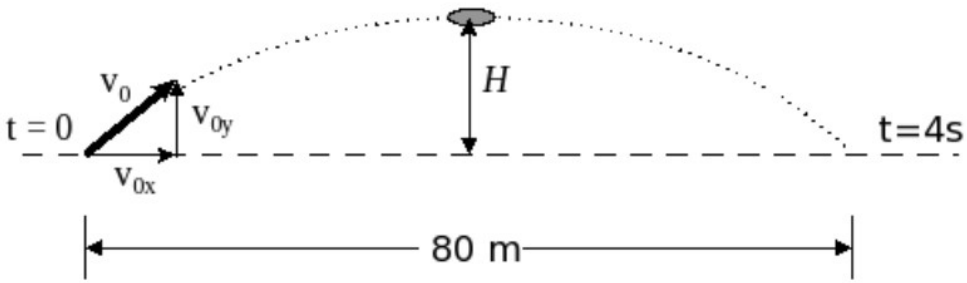
Final Exam Topics

- Section 1
  - Vectors
  - Kinematics (incl polar coordinates)
  - Newton's Laws
  - Friction and Springs
  - Harmonic Oscillator
- Section 2
  - Work-energy theorem
  - Energy conservation, conservative forces
  - Center-of-Mass, Momentum Conservation
  - Lagrangian Mechanics
  - Rotational Dynamics
- Section 3
  - Angular Momentum Vector
  - Fictitious Forces
  - Central Forces, Planetary Motion
  - ~~Damped Oscillator~~
  - Special Relativity

Not yet covered in exam!!!

2D Kinematics: Projectile Motion

2. (16 pts) A football is kicked as shown in the figure. It flies for 4.00 seconds, and travels 80.0 m horizontally before it lands. Neglect the effects of air resistance in answering the questions that follow.



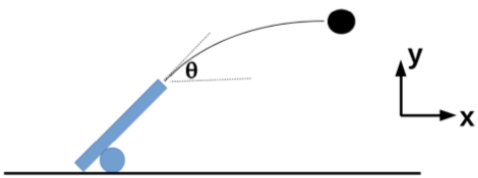
(a) (4) What is the speed of the ball at the top of its trajectory?

(b) (6) What is the maximum height H reached by the ball?

(c) (6) What is the speed v_o of the ball immediately after it is kicked?

- Independent horizontal and vertical motion
  - Horizontal: constant velocity
  - Vertical: constant acceleration

4. (16 pts) A cannon fires a projectile with initial speed v at an angle of θ with respect to horizontal. The projectile flies under the influence of uniform gravity, before returning to Earth. Assume air resistance is negligible.



Indicate with a "T" or "F" whether the following statements are true or false while the projectile is in flight:

(a) _____ The y component of the projectile's velocity is never zero.
(b) _____ The horizontal (x) component of the projectile's velocity remains constant.
(c) _____ The acceleration of the projectile remains constant during its upward flight.
(d) _____ The vertical (y) component of the projectile's velocity remains constant.
(e) _____ The horizontal acceleration of the projectile is zero.
(f) _____ The vertical acceleration of the projectile is zero.
(g) _____ The x component of the projectile's velocity is never zero.
(h) _____ The minimum speed of the projectile during its flight is equal to $v \sin\theta$.

Linear Kinematics		Rotational Kinematics	
Displacement	x	θ	Angular displacement
Velocity	$v \equiv \dfrac{dx}{dt}$	$\omega \equiv \dfrac{d\theta}{dt}$	Angular velocity
Acceleration	$a \equiv \dfrac{dv}{dt}$	$\alpha \equiv \dfrac{d\omega}{dt}$	Angular acceleration

For constant acceleration	For constant angular acceleration
$x = x_o + v_o t + \frac{1}{2}at^2$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$v^2 - v_o^2 = 2a(x - x_o)$	$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$

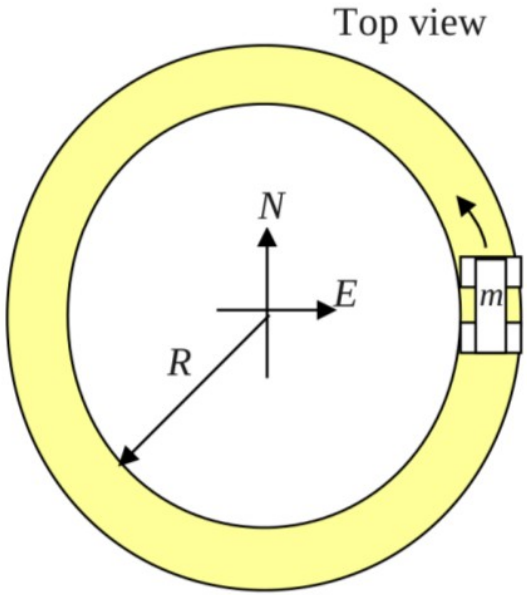
Newton's Laws

1) An object subject to no net external force is at rest or moving at a constant velocity when viewed from an *inertial reference frame*.

2) $\mathbf{a} = \mathbf{F}_{net}/m$

3) For every action there is an equal and opposite reaction $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$

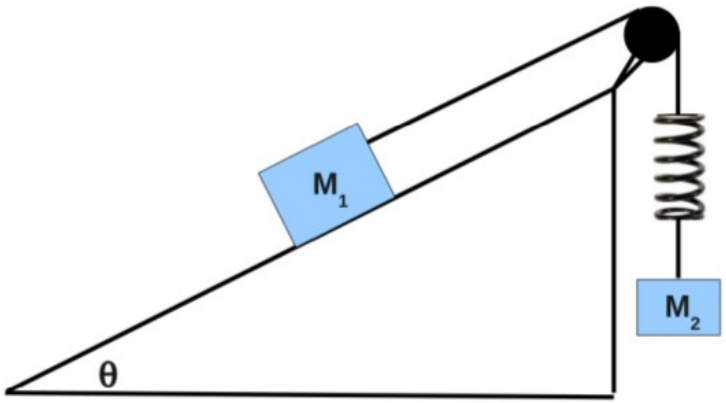
3. (4 pts) A car is driving on a circular track as shown in the figure below. At the time shown, the driver steps on brakes so that the car's tangential speed is decreasing. On the figure below, sketch the direction of the net frictional force being applied by the car's wheels on the track. The wheels are rolling without slipping.



Forces and Free-Body Diagrams

- FBD: Tool for generating equations for dynamic systems
- Each object in the system is drawn *isolated from all but the forces acting on it*.

7. (22 pts) Two masses are connected as shown in the figure below. $M_1 = 4.0$ kg slides on a frictionless ramp making an angle of $\theta = 22°$ with respect to the horizontal. M_1 is connected to $M_2 = 3.0$ kg via a massless, frictionless pulley and a massless spring. The spring constant of the spring is $k = 100.0$ N/m.



(a) (6) Draw the free body diagrams of masses M_1 and M_2.

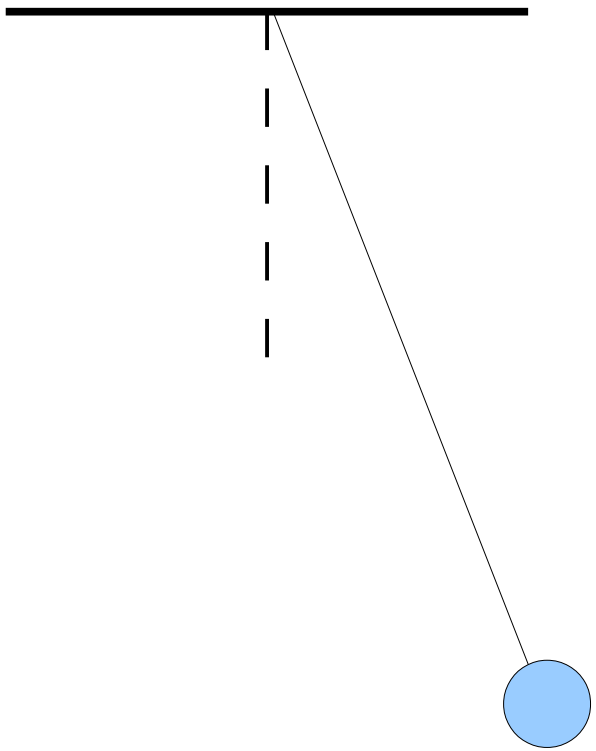
b) what is the acceleration (magnitude and direction) of m_2?

c) what is the amount by which the spring is stretched from equilibrium?

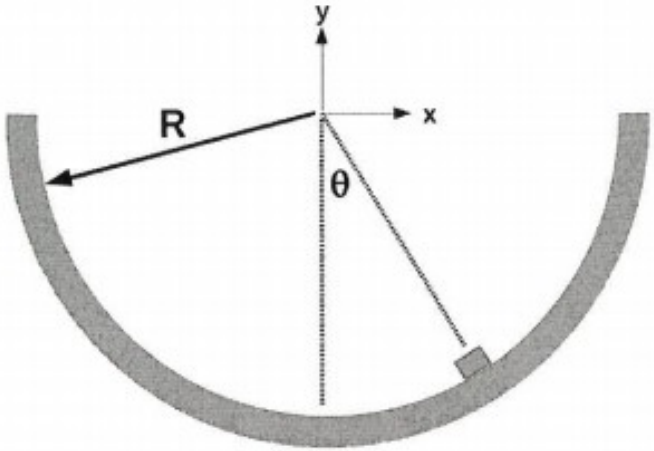
Simple Pendulum/Harmonic Motion

The simple pendulum shown - consisting of a massless string of length L = 1.0 m and a bob of mass 0.1 kg - is given an initial velocity v_0 = 3.0 m/s to the right, at an initial displacement θ_0 = 0. Find:

- The position of the pendulum at t = 8 seconds.
- The tension in the string at t = 8 seconds.



5. (14 pts) A small block of ice slides without friction in a spherical bowl of radius $R = 0.2$ m. Suppose the motion of the block is confined to the xy plane. It should be clear from the diagram below that the block's motion will be similar to that of the bob of a simple pendulum, with the normal force of the bowl on the ice playing the role of the tension in a simple pendulum's string.



Making the small angle approximation $\sin\theta \approx \theta$, and noting that $x = R\sin\theta \approx R\theta$, one can show that the x coordinate of the block will obey the differential equation

$$\frac{d^2x(t)}{dt^2} + \frac{g}{R}x(t) = 0$$

Final Exam Topics

- Section 1
  - Vectors
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- Section 2
  - Work-energy theorem
  - Energy conservation, conservative forces
  - Center-of-Mass, Momentum Conservation
  - Lagrangian Mechanics
  - Rotational Dynamics
- Section 3
  - Angular Momentum Vector
  - Fictitious Forces
  - Central Forces, Planetary Motion
  - Damped Oscillator
  - Special Relativity

Not yet covered in exam!!!

Work-Kinetic Energy Theorem

$$W_{net} = \Delta K$$

The net work done on a body is equal to the change in kinetic energy of the body

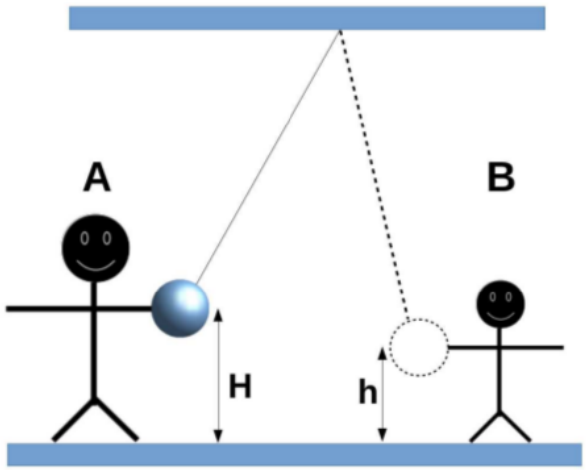
$$W = \int_{\vec{r_1}}^{\vec{r_2}} \vec{F} \cdot \vec{dl}$$

Formal definition of work (“Force times distance” generalized)

$$K = \frac{1}{2} m v^2$$

Formal definition of kinetic energy

1. (8 pts) Alice (A) is holding a pendulum bob of mass m at a height H above the ground (solid line). At time $t = t_o$, she releases the bob from rest, and it swings until caught and brought to rest by Brad (B) at a lesser height h at time $t = t_1$ (dashed line). The following questions pertain to the work done on the bob between t_0 and t_1 only. Be sure to give both magnitudes and signs in your answers.



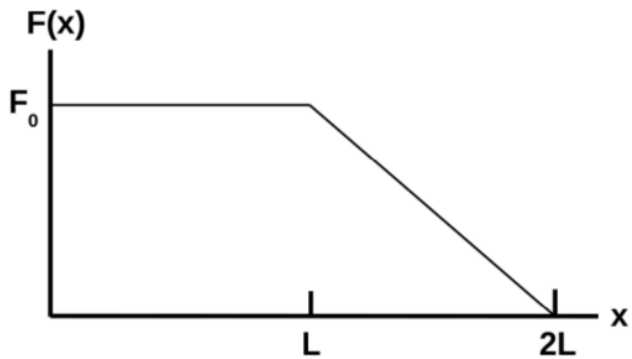
(a) (2) What work did Alice do on the bob? _____

(b) (2) What work did gravity do on the bob? _____

(c) (2) What work did tension do on the bob? _____

(d) (2) What work did Brad do on the bob? _____

2. (8 pts) The graph below shows the force applied in the positive x-direction, to an object of mass m as a function of position x. The object starts at rest. The force is constant F_0 until the object reaches $x = L$, then it decreases linearly to zero until the object reaches $x = 2L$. What is the speed of the object at $x = 2L$?



Generalize mechanical energy conservation to conservative systems including springs:

$$\frac{1}{2}kx_i^2 + mgh_i + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + mgh_f + \frac{1}{2}mv_f^2$$

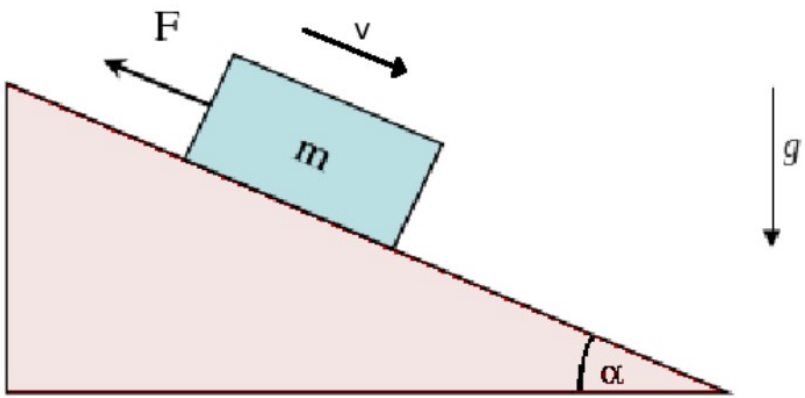
Spring P.E.

Gravitational P.E.

K.E.

Work-Energy Theorem

8. (24 pts) A block of mass $m = 105.0$ kg, starting from rest, slides down a ramp making an angle $\alpha = 32°$ with respect to horizontal. The ramp is 5.0 meters long. A constant frictional force of 190.0 N acts throughout the motion, and a force F is being applied by a rope as shown in order to prevent the block from sliding too fast.



(a) (6) Calculate the work done by the frictional force if the block travels the length of the ramp.

b) If the force f is such that the block will move with constant speed v = 0.1 m/s down the ramp, calculate the work done by the force F if the block travels the length of the ramp.

c) Suppose that, while traveling at v = 0.1 m/s down the ramp, at d = 3 meters from the end of the ramp the rope breaks. Use the work-kinetic energy theorem to calculate the speed of the crate when it reaches the bottom of the ramp.

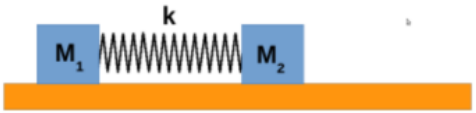
Lagrangian Mechanics

Basic program:

L = T – V

Apply Euler-Lagrange eqns

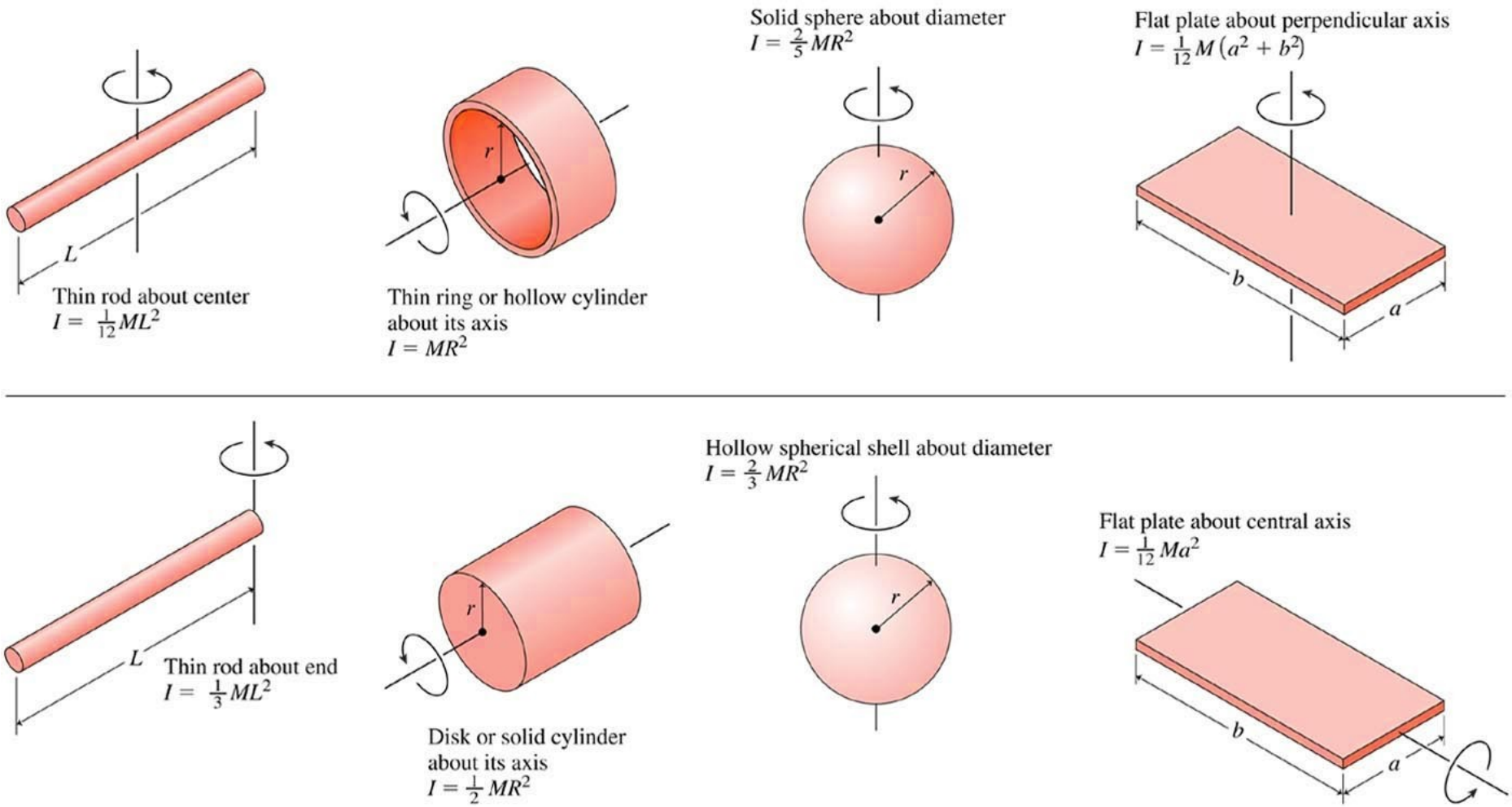
3. (14 pts) Two blocks with mass M_1 and M_2 are connected by a spring with spring constant k and are sliding on a frictionless horizontal surface. Their positions are given by coordinates x_1 and x_2 respectively, defined such that when $x_1 = x_2$ the spring is unstretched.



(a) (4) Write down the Lagrangian for this system.

(b) (4) Use the Lagrangian to determine two coupled equations of motion for the two blocks.

(c) (6) Show that if M_1 is moving with constant velocity then M_2 must also be moving with constant velocity.



You will be given I for basic shapes.

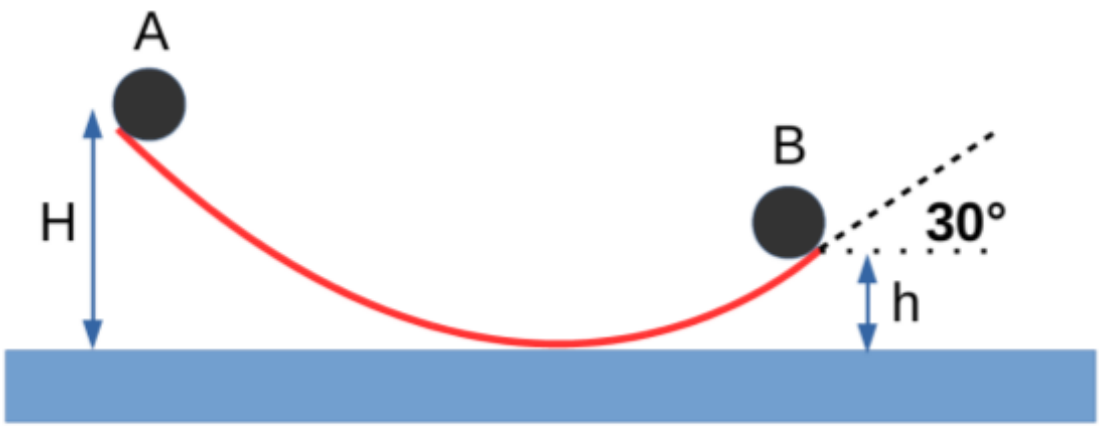
$$\text{Mechanical Energy} = \underbrace{\frac{1}{2}kx^2}_{\text{Spring PE}} + \underbrace{mgh}_{\text{Gravitational PE}} + \underbrace{\frac{1}{2}mv^2}_{\text{Translational KE}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{Rotational KE}}$$

$I = \Sigma\, m_i r_i^2$

= "moment of inertia"

Rotational Dynamics

7. (16 pts) A solid disk of radius R and mass m is released from rest at Point A on a curved ramp, at a height H above the bottom. The disk rolls without slipping to B, at which point it leaves the ramp at an angle of 30° with respect to horizontal.

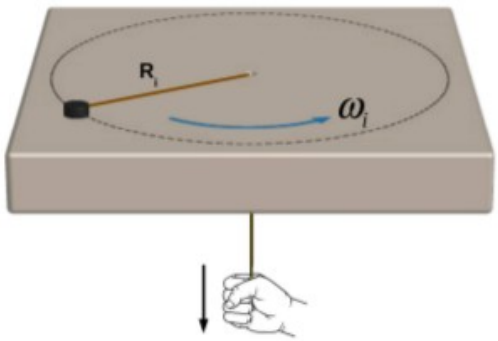


(a) (10) In terms of m, g, R, H and h, what is the *angular* speed of the disk at point B?

Similarity to 1D motion

Linear Motion		Rotational Motion
$x = x_o + v_o t + \frac{1}{2}at^2$	$\longleftrightarrow$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
$v = v_o + at$	$\longleftrightarrow$	$\omega = \omega_o + \alpha t$
m	$\longleftrightarrow$	I
$\frac{1}{2}mv^2$	$\longleftrightarrow$	$\frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$	$\longleftrightarrow$	$\vec{\tau} = I\vec{\alpha}$

1. (20 pts) See the figure below. A rectangular block of ice has a hole cut through it. A hockey puck slides without friction on the ice, while attached to a string which passes through the hole. Initially, the block is a distance R_i from the hole, moving with an angular velocity ω_i around the hole. A hand pulls the string downwards as shown.



(a) (5) If the hand pulls the string until the puck is only one third as far from the hole (that is, $R_f = R_i/3$), what is the ratio of final to initial *angular* speed of the puck?

(b) (5) What is the ratio of final to initial *tangential* speed of the puck?

(c) (5) What is the ratio of final to initial *angular momentum* of the puck?

(d) (5) What is the ratio of final to initial *kinetic energy* of the puck?

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  - Special Relativity

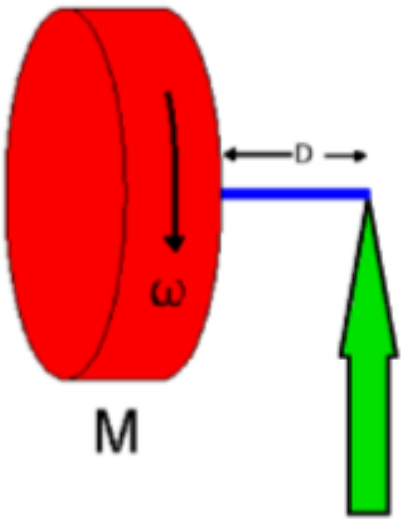
Not yet covered in exam!!!

2. (12 pts) Consider the gyroscope shown in the figure. A disk of mass M rotates upon its axis with angular speed ω in the direction shown, a distance D from its point of support. Indicate the direction of the following from your viewing perspective as "left", "right", "into the page" or "out of the page":

(a) (4) Angular momentum vector _____

(b) (4) Torque due to gravity _____

(c) (4) Precession of gyroscope _____

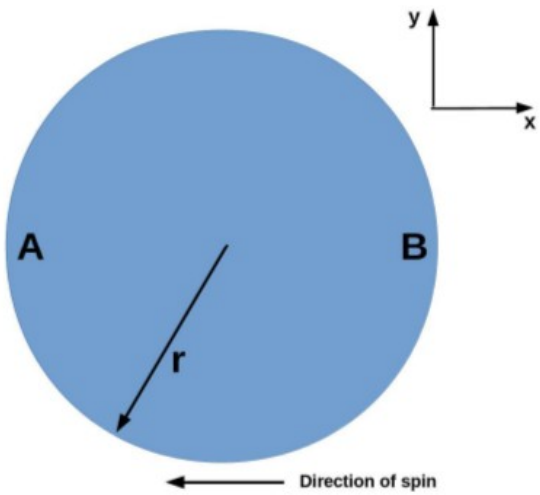


Fictitious Forces

$$m\vec{a} = \vec{F} + \vec{F}_{translation} + \vec{F}_{centrifugal} + \vec{F}_{coriolis} + \vec{F}_{azimuthal}$$

$$\begin{aligned}
\vec{F}_{translation} &= -m\frac{d^2\vec{R}}{dt^2} \\
\vec{F}_{centrifugal} &= -m\vec{\omega} \times (\vec{\omega} \times \vec{r}) \\
\vec{F}_{coriolis} &= -2m(\vec{\omega} \times \vec{v}) \\
\vec{F}_{azimuthal} &= -m\frac{d\vec{\omega}}{dt} \times \vec{r}
\end{aligned}$$

9. (14 pts) Annie (A) and Bob (B) stand on opposite sides of a spinning carousel of radius $r = 3.0$ m. Annie's mass is 60 kg and Bob's is 80 kg, and the mass of the carousel is 400 kg. The carousel spins clockwise at $(1/4\pi)$ revolutions per second. The x and y-directions in the rotating carousel frame are as indicated. The z-direction is out of the page.



(a) (4) What are the magnitude and direction of the fictitious "centrifugal force" on Annie? What is the magnitude and direction of the non-fictitious centripetal force on her feet? (Problem continues on the next page.)

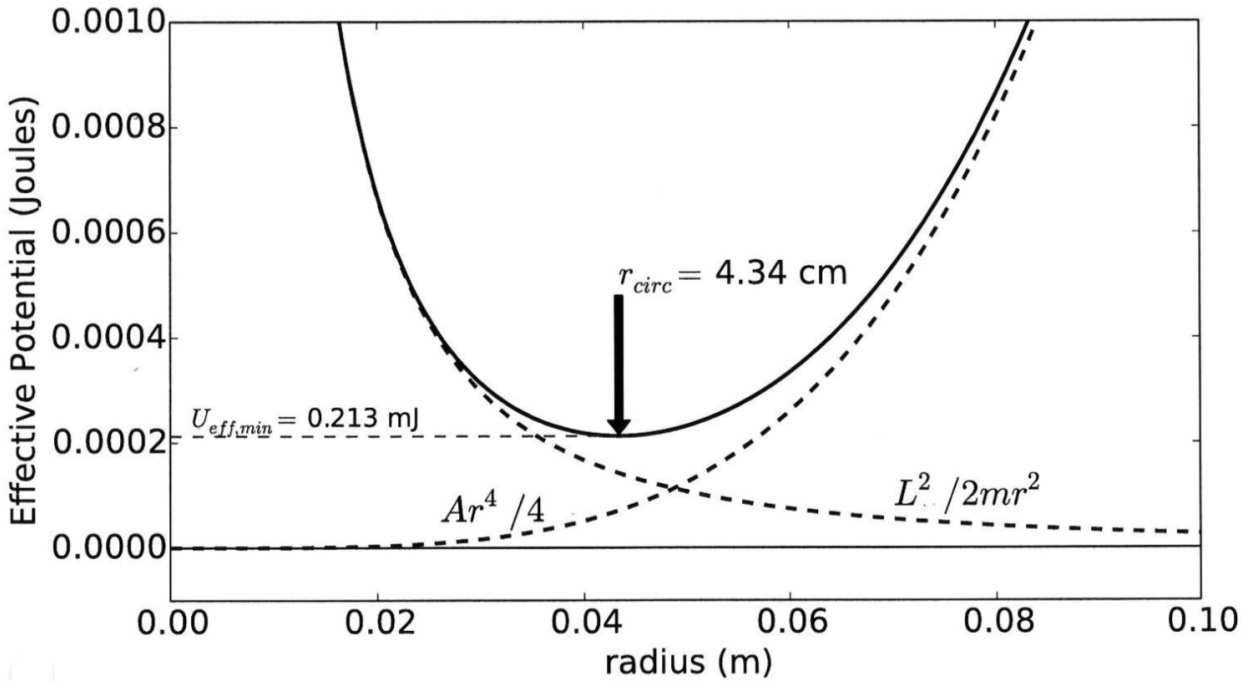
(b) If Annie walks to the center of the carousel (and Bob stays where he is), what is the final rate of spin of the carousel?

(c) What is the direction of the Coriolis force on Annie as she walks?

(d) What is the direction fo the azimuthal force on Bob as Annie walks?

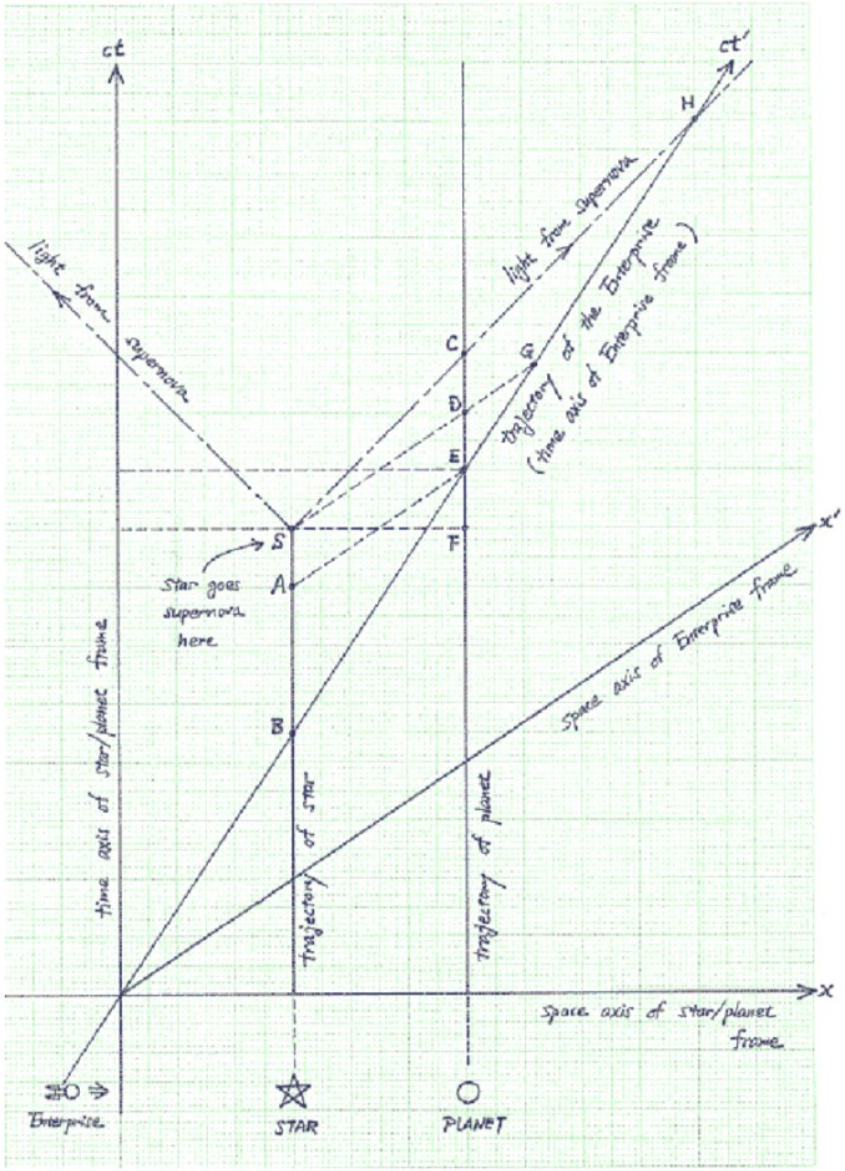
Potentials, Central Forces

- For conservative forces, the force is the spatial derivative of a potential function.
  - e.g. $F(x) = - dU(x)/dr$
- Orbital motion may be described by an effective potential
  - $U_{eff} = L^2/2mr^2 + U(r)$



Special Relativity

- Origins
  - Michelson-Morley expt
  - What problem did SR solve?
- The invariant interval
  - $ds^2 = -c^2dt^2 + dx^2$
- Time dilation and length contraction
- Expect: Workout with space-time diagram.



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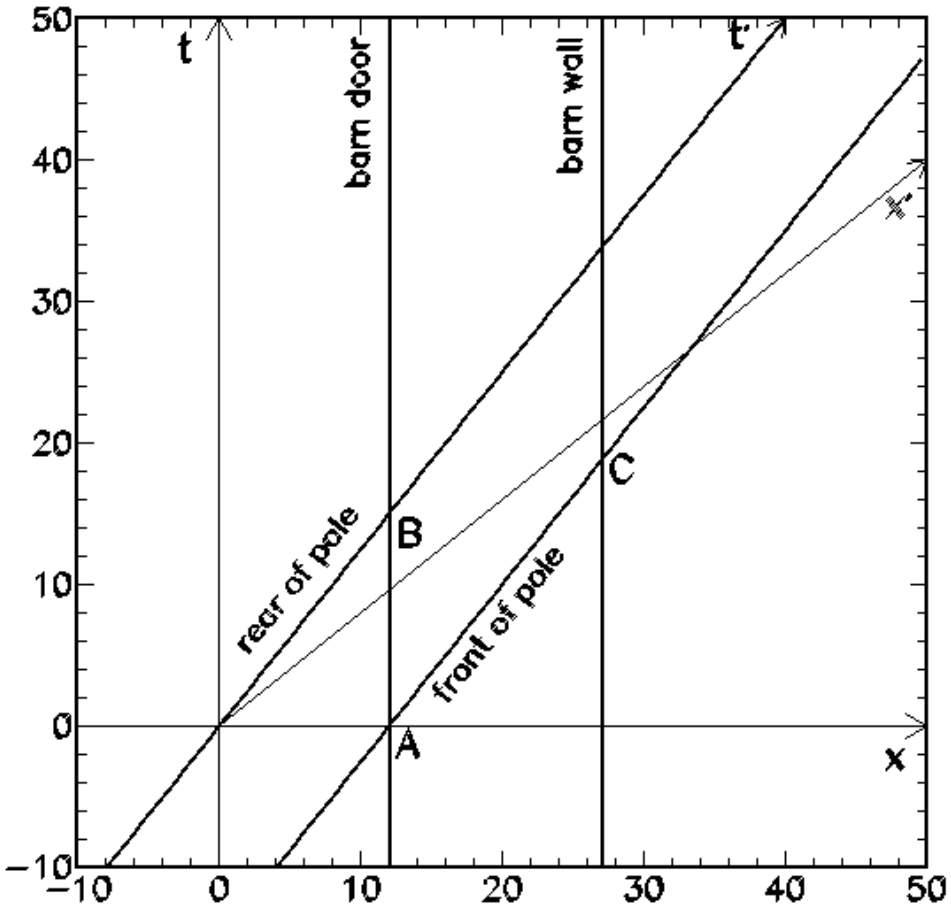
  - Michelson-Morley expt
  - What problem did SR solve?
- The invariant interval
  - $ds^2 = -c^2dt^2 + dx^2$
- Time dilation and length contraction
- Expect: Workout with space-time diagram.



Final Exam Topics

- Section 1
  - Vectors
  - Kinematics (incl polar coordinates)
  - Newton's Laws
  - Friction and Springs
  - Harmonic Oscillator
- Section 2
  - Work-energy theorem
  - Energy conservation, conservative forces
  - Center-of-Mass, Momentum Conservation
  - Lagrangian Mechanics
  - Rotational Dynamics
- Section 3
  - Angular Momentum Vector
  - Fictitious Forces
  - Central Forces, Planetary Motion
  - Damped Oscillator
  - Special Relativity

Not yet covered in exam!!!