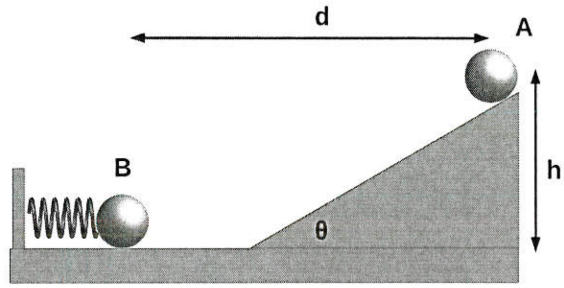
Physics 3210, Spring 2019
Exam #2

Name: Solutions
Signature:
UID:

Please read the following before continuing:

- Show all work in answering the following questions. Partial credit may be given for problems involving calculations.
- Be sure that your final answer is clearly indicated, for example by drawing a box around it.
- Be sure that your cellphone is turned off.
- Your signature above indicates that you have neither given nor received unauthorized assistance on any part of this exam.
- Thanks, and good luck!

1. (12 pts) A ball of radius R and mass m is released at rest from position **A** at the top of a ramp, making an angle θ with respect to horizontal. The ball starts at a height **h** above the ground. It rolls without slipping down the ramp, then compresses a spring with constant **k**. At position **B** the spring is at maximum compression. Answer the questions below regarding the work done on the ball between positions **A** and **B**.



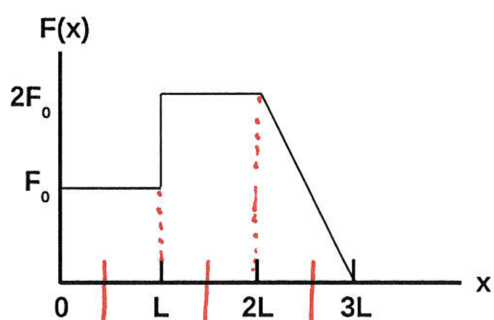
(a) (3) What work did gravity do on the ball? $+mgh$

(b) (3) What work did the normal force of the ramp do on the ball? 0

(c) (3) What work did the spring do on the ball? $-mgh$

(d) (3) What is the net work done on the ball? 0

2. (10 pts) The graph below shows the force applied in the positive x-direction, to an object of mass m as a function of position x. The object starts with speed v_0 in the positive x-direction. The force is constant F_0 until the object reaches $x = L$, it is doubled between $x = L$ and $2L$, then it decreases linearly to zero until the object reaches $x = 3L$. What is the speed of the object at $x = 3L$?



Work $= \int \vec{F} \cdot d\vec{x}$

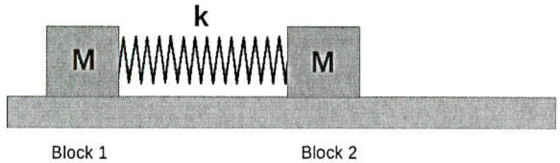
$F_0 L$ $\quad 2F_0 L$ $\quad \frac{1}{2}(2F_0 L)$ $\qquad$ Total work $= +4F_0 L$

Work - Energy Theorem $\qquad$ Work $= \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_0^2 = 4F_0 L + \frac{1}{2}mv_0^2$$

$$\boxed{v_f = \sqrt{\frac{2}{m}\left(4F_0 L + \frac{1}{2}mv_0^2\right)}}$$

3. (18 pts) Two blocks with equal masses M are connected by a spring with spring constant k and are sliding on a frictionless horizontal surface. Their positions are given by coordinates x_1 and x_2 respectively, defined such that when $x_1 = x_2$ the spring is unstretched.



(a) (6) Write down the Lagrangian for this system.

$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 \qquad U = \frac{1}{2} k (x_1 - x_2)^2$

$$\boxed{\mathcal{L} = T - U = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k (x_1 - x_2)^2}$$

(b) (6) Use the Lagrangian to determine two coupled equations of motion for the two blocks.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_1} - \frac{\partial \mathcal{L}}{\partial x_1} = \boxed{m \ddot{x}_1 + k(x_1 - x_2) = 0} \quad \text{i}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} - \frac{\partial \mathcal{L}}{\partial x_2} = \boxed{m \ddot{x}_2 - k(x_1 - x_2) = 0} \quad \text{ii}$$

(c) (6) Find the frequency of small oscillations of this system. *Hint: combine the equations such that by making the variable change $y = x_1 - x_2$ you obtain the simple harmonic oscillator equation.*

SUBTRACT EQN ii from EQN i:

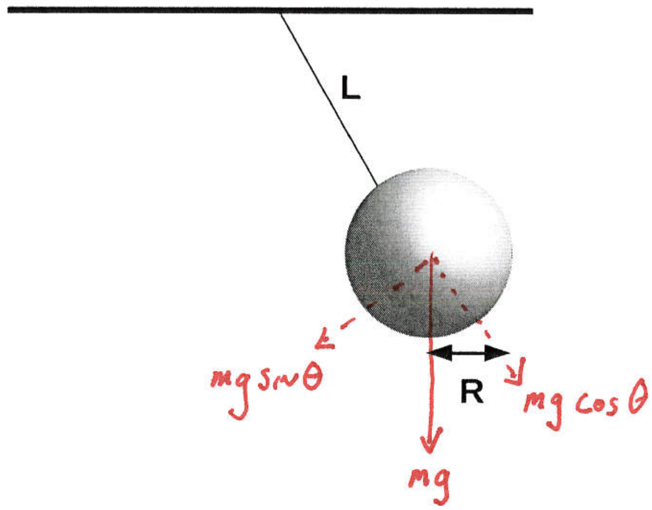
$$m(\ddot{x}_1 - \ddot{x}_2) + 2k(x_1 - x_2) = 0$$

Let $y = x_1 - x_2$

$$m\ddot{y} + 2ky = 0$$

$$\ddot{y} + \frac{2k}{m} y = 0 \longrightarrow \text{SHO WITH } \boxed{\omega = \sqrt{\frac{2k}{m}}}$$

4. (14 pts) A pendulum consists of a sphere of mass m and radius R on the end of a massless string of length L.



(a) (8) *Without treating the sphere as pointlike*, calculate the moment of inertia of the pendulum about the axis of rotation.

$$I_{Axis} = I_{Sphere,\, cm} + M r_{cm}^2$$ Parallel Axis Theorem

$$\boxed{I_{Axis} = \frac{2}{5} mR^2 + m(L+R)^2}$$

(b) (6) What is the period of small oscillations of this pendulum?

$$\Sigma \tau = -mg \sin\theta\,(L+R) = I\alpha = I\ddot{\theta}$$ small angle, $\sin\theta \approx \theta$

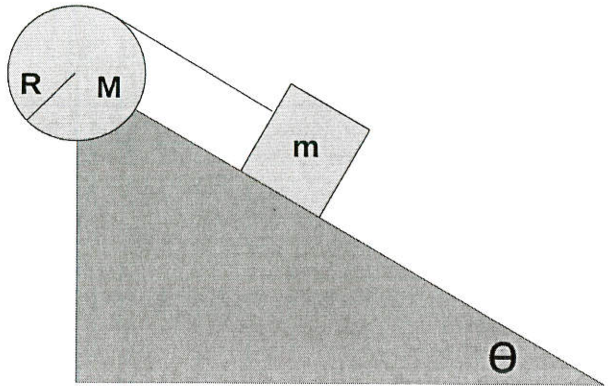
$$\ddot{\theta} + \underbrace{\frac{mg(L+R)}{I}}_{=\omega}\theta = 0$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mg(L+R)}}$$

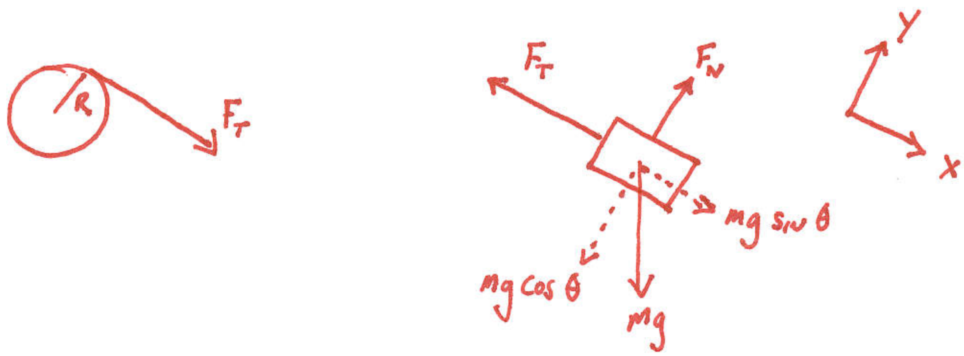
$$\boxed{T = 2\pi\sqrt{\frac{\frac{2}{5}R^2 + (L+R)^2}{g(L+R)}}}$$

5. (12 pts) A block of mass m, on a frictionless ramp making an angle θ to the horizontal, hangs from a massless string. The string is wound around a solid drum of radius R and mass M. The drum is free to rotate about its center.

If the string unwinds without slipping, what is the acceleration of the hanging block in terms of m, M, R, θ and g?



FBD



$$\Sigma \tau = -F_T R = I\alpha$$

$$I = \frac{1}{2}MR^2 \quad \text{(disk)}$$

$$\alpha = -\frac{a_x}{R} \quad \text{(no slipping)}$$

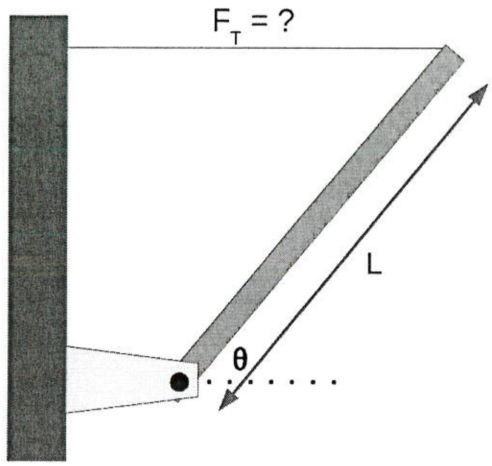
$$\hookrightarrow F_T = \frac{1}{2}Ma_x$$

$$\Sigma F_x = mg\sin\theta - F_T = ma_x$$

$$mg\sin\theta = \left(\frac{M}{2} + m\right)a_x$$

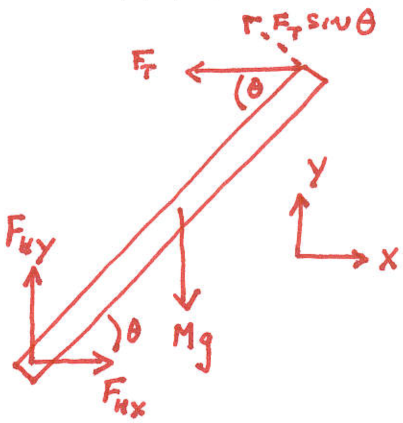
$$\boxed{a_x = \frac{mg\sin\theta}{\left(\frac{M}{2} + m\right)}}$$

6. (22 pts) A rod of length L and mass M is connected to a wall by a hinge as shown in the figure. A horizontal string supports the far end of the rod, so that it makes an angle θ with respect to horizontal.



(a) (12) What is the tension F_T in the horizontal string?

FBD



$$\Sigma \tau = F_T L \sin\theta - Mg\left(\frac{L}{2}\right)\cos\theta = 0$$

$$\boxed{F_T = \frac{1}{2} Mg \frac{\cos\theta}{\sin\theta} = \frac{1}{2} Mg \cot\theta}$$

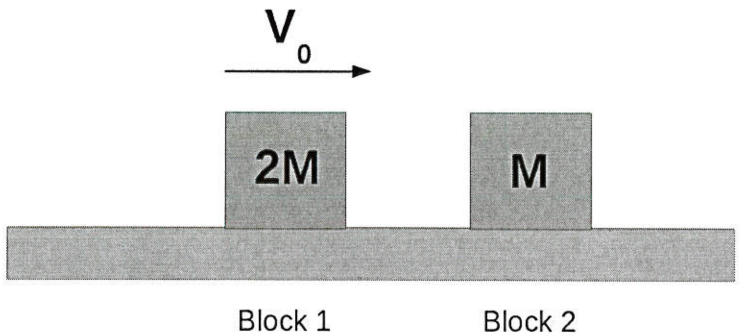
(b) (10) What is the magnitude of the force exerted by the hinge on the rod?

$$\Sigma F_x = F_{Hx} - F_T = 0; \quad F_{Hx} = F_T = \frac{1}{2} Mg \cot\theta$$

$$\Sigma F_y = F_{Hy} - Mg = 0; \quad F_{Hy} = Mg$$

$$F_H = \sqrt{F_{Hx}^2 + F_{Hy}^2} = \boxed{Mg\sqrt{\frac{1}{4}\cot^2\theta + 1}}$$

7. (12 pts) Two blocks, as shown in the figure, are on a frictionless surface. Block 1 with mass $2M$ slides to the right with speed v_0, until it collides elastically with a Block 2 of mass M which is initially at rest. Calculate the final velocities (magnitude and direction) of the two blocks, v_1 and v_2.



Energy Conservation

$$\frac{1}{2}(2M)V_0^2 = \frac{1}{2}(2M)V_1^2 + \frac{1}{2}MV_2^2$$

$$V_0^2 = V_1^2 + \frac{1}{2}V_2^2$$

$$V_1 = \sqrt{V_0^2 - \frac{1}{2}V_2^2}$$

Momentum Conservation

$$2mV_0 = 2mV_1 + mV_2$$

$$V_0 = V_1 + \frac{1}{2}V_2$$

$$V_1 = V_0 - \frac{1}{2}V_2$$

Some arithmetic ...

$$V_0 - \frac{1}{2}V_2 = \sqrt{V_0^2 - \frac{1}{2}V_2^2}$$

$$\cancel{V_0^2} + \frac{1}{4}V_2^2 - V_0V_2 = \cancel{V_0^2} - \frac{1}{2}V_2^2$$

$$-V_0V_2 = -\frac{3}{4}V_2^2$$

$$\boxed{V_2 = \frac{4}{3}V_0, \text{ TO RIGHT}}$$

$$V_1 = \sqrt{V_0^2 - \frac{1}{2}V_2^2} = \sqrt{V_0^2 - \frac{1}{2}\frac{16}{9}V_0^2} = \sqrt{\frac{9}{9} - \frac{8}{9}}\,V_0$$

$$\boxed{V_1 = \frac{1}{3}V_0, \text{ TO RIGHT}}$$