Physics 3210, Spring 2018

Exam #2

Name: Solutions

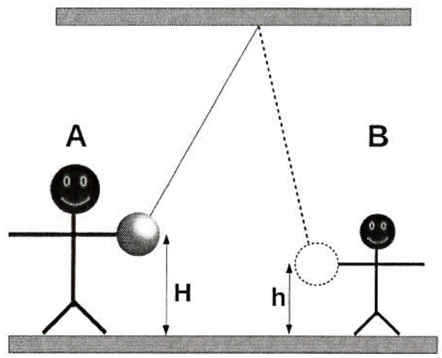
Signature:

UID:

Please read the following before continuing:

- Show all work in answering the following questions. Partial credit may be given for problems involving calculations.
- Be sure that your final answer is clearly indicated, for example by drawing a box around it.
- Be sure that your cellphone is turned off.
- Your signature above indicates that you have neither given nor received unauthorized assistance on any part of this exam.
- Thanks, and good luck!

1. (8 pts) Alice (A) is holding a pendulum bob of mass m at a height H above the ground (solid line). At time $t = t_o$, she releases the bob from rest, and it swings until caught and brought to rest by Brad (B) at a lesser height h at time $t = t_1$ (dashed line). The following questions pertain to the work done on the bob between t_0 and t_1 only. Be sure to give both magnitudes and signs in your answers.



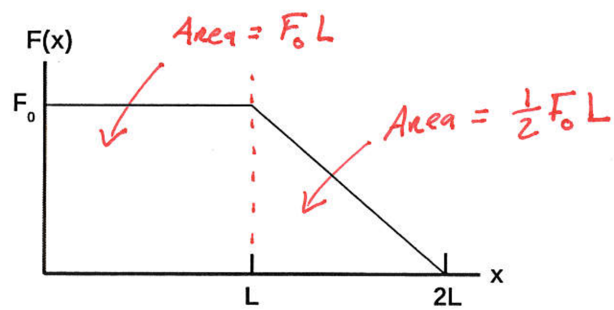
   (a) (2) What work did Alice do on the bob? $\varnothing$

   (b) (2) What work did gravity do on the bob? $mg(H-h)$

   (c) (2) What work did tension do on the bob? $\varnothing$

   (d) (2) What work did Brad do on the bob? $-mg(H-h)$

2. (8 pts) The graph below shows the force applied in the positive x-direction, to an object of mass m as a function of position x. The object starts at rest. The force is constant F_0 until the object reaches $x = L$, then it decreases linearly to zero until the object reaches $x = 2L$. What is the speed of the object at $x = 2L$?



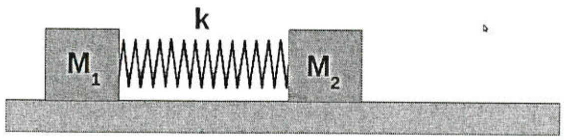
WORK - ENERGY THEOREM: WORK = Δ K.E.

WORK DONE = $\int F \cdot dx = F_0 L + \frac{1}{2} F_0 L = \frac{3}{2} F_0 L$

$\Delta KE = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}$ = 0, STARTS AT REST

$\frac{3}{2} F_0 L = \frac{1}{2} m v_f^2$; $\boxed{v_f = \sqrt{\frac{3 F_0 L}{m}}}$

3. (14 pts) Two blocks with mass M_1 and M_2 are connected by a spring with spring constant k and are sliding on a frictionless horizontal surface. Their positions are given by coordinates x_1 and x_2 respectively, defined such that when $x_1 = x_2$ the spring is unstretched.



(a) (4) Write down the Lagrangian for this system.

$$\mathcal{L} = K - U = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} k (x_1 - x_2)^2$$

(b) (4) Use the Lagrangian to determine two coupled equations of motion for the two blocks.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_1} - \frac{\partial \mathcal{L}}{\partial x_1} = m_1 \ddot{x}_1 + k(x_1 - x_2) = 0$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} - \frac{\partial \mathcal{L}}{\partial x_2} = m_2 \ddot{x}_2 - k(x_1 - x_2) = 0$$

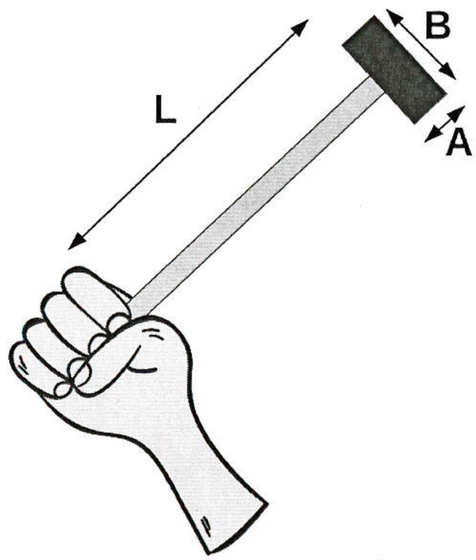
(c) (6) Show that if M_1 is moving with constant velocity then M_2 must also be moving with constant velocity.

ADD THE TWO EQUATIONS: $m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0$

$$\ddot{x}_2 = -\frac{m_1}{m_2} \ddot{x}_1$$

IF $\ddot{x}_1 = 0$ (m_1 CONSTANT VELOCITY) THEN $\ddot{x}_2 = 0$ QED

4. (6 pts) A carpenter swings a hammer to strike a nail. The hammer consists of a handle (thin rod) of mass m and length L, and a rectangular head of width A, length B and mass M. Calculate the moment of inertia of the hammer, about the end of the hammer in the carpenter's hand.



$$I_{HAMMER} = I_{ROD} + I_{HEAD}$$

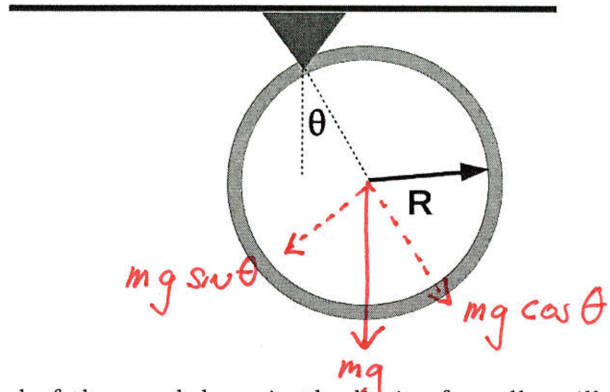
$$I_{ROD} = \frac{1}{3} m L^2 \qquad \text{(TABLE)}$$

$$I_{HEAD} = I_{cm} + M R_{cm}^2 \qquad \text{(PARALLEL AXIS THEOREM)}$$

$$= \frac{1}{12} M (A^2 + B^2) + M (L + A/2)^2$$

$$\boxed{I_{HAMMER} = \frac{1}{3} m L^2 + \frac{1}{12} M (A^2 + B^2) + M (L + A/2)^2}$$

5. (16 pts) A pendulum consists of a thin ring of mass M and radius R, attached to a pivot at a point on its rim.



(a) (8) Compute the period of the pendulum, in the limit of small oscillations.

$$I_{RING} = I_{RING,CM} + MR_{cm}^2 \text{ (PARALLEL AXIS THEOREM)} = MR^2 + MR^2 = \underline{\underline{2MR^2}}$$

$$\sum \tau = Mg R \sin\theta = I\alpha = -2MR^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{1}{2}\frac{g}{R}\sin\theta = 0$$

SMALL OSCILLATIONS

$$\ddot{\theta} + \frac{1}{2}\frac{g}{R}\theta = 0$$

THIS IS SHO EQUATION WITH $\omega = \sqrt{\frac{g}{2R}}$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2R}{g}}}$$

Problem continued on the following page.

(b) (8) Write a particular solution for the pendulum's motion in the case that it is initially hanging vertically and has initial angular velocity ω_0 clockwise.

GENERAL SOLUTION $\theta(t) = A\cos(\omega t + \phi)$

$\dot{\theta}(t) = -\omega A \sin(\omega t + \phi)$

INITIAL CONDITIONS $\theta(0) = A\cos\phi = 0 \rightarrow \underbrace{\phi = \pi/2}$

$\dot{\theta}(0) = -\omega A \sin(\pi/2) = -\omega A = -\omega_0$

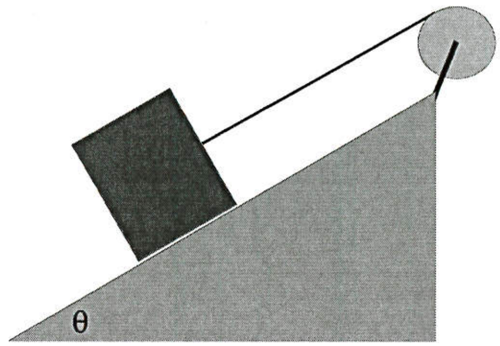
CLOCKWISE

$$\underbrace{A = \frac{\omega_0}{\omega} = \omega_0\sqrt{\frac{2R}{g}}}$$

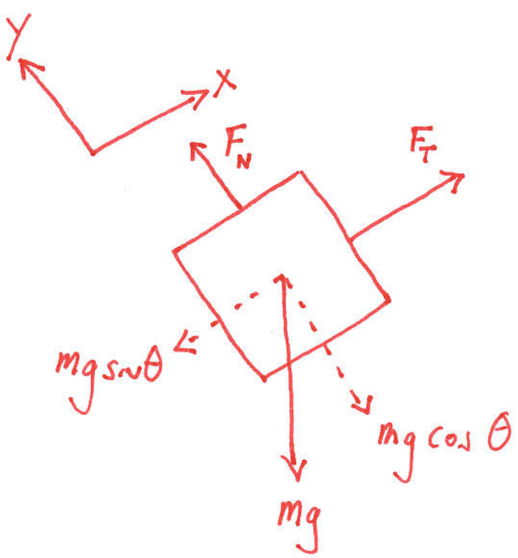
FINALLY,

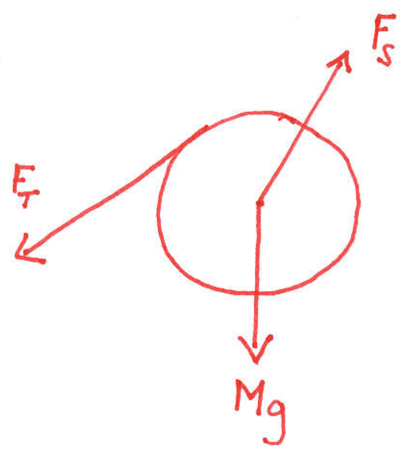
$$\boxed{\theta(t) = \omega_0\sqrt{\frac{2R}{g}}\cos\left(\sqrt{\frac{2R}{g}}\,t + \frac{\pi}{2}\right)}$$

6. (16 pts) A block of mass m is held by a long massless string on a frictionless inclined plane inclined at an angle θ to the horizontal. The string is wound on a uniform solid cylindrical drum of mass M and radius R as shown in the figure. The drum is given an initial angular speed ω such that the block starts moving up the plane.



(a) (8) Draw complete *Free Body Diagrams* for the block and drum. Be sure to indicate *all* forces acting on each.





Problem continued on the following page.

(b) (8) Find the tension in the string during the motion.

ON BLOCK $\sum F_x = F_T - mg\sin\theta = ma_x$

ON DRUM $\sum \tau = F_T R = I\alpha = -\left(\frac{1}{2}MR^2\right)\frac{a_x}{R}$

$$a_x = -\frac{2F_T}{M}$$

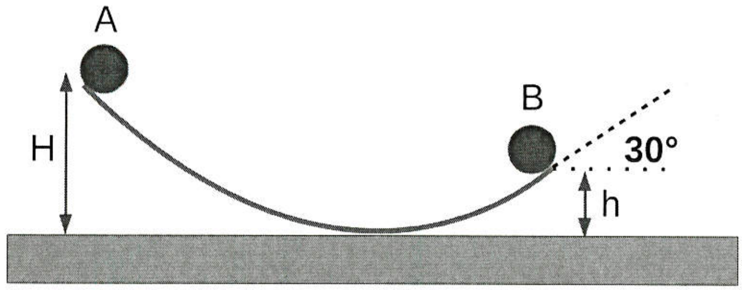
PLUG INTO BLOCK EQN

$$F_T = mg\sin\theta + ma_x$$

$$= mg\sin\theta - m\cdot\frac{2F_T}{M}$$

$$\boxed{F_T = \frac{mg\sin\theta}{\left(1 + \frac{2m}{M}\right)}}$$

7. (16 pts) A solid disk of radius R and mass m is released from rest at Point A on a curved ramp, at a height H above the bottom. The disk rolls without slipping to B, at which point it leaves the ramp at an angle of 30° with respect to horizontal.



(a) (10) In terms of m, g, R, H and h, what is the *angular* speed of the disk at point B?

$$mgH = mgh + \frac{1}{2}mv_B^2 + \frac{1}{2}I\omega_B^2$$

$I = \frac{1}{2}mR^2$ (disk) $\quad \omega = V/R$ (roll without slipping)

$$mgH = mgh + \frac{1}{2}mv_B^2 + \frac{1}{4}mv_B^2$$

$$gH = gh + \frac{3}{4}v_B^2$$

$$V_B = \sqrt{\frac{4g(H-h)}{3}} \longrightarrow \boxed{\omega_B = \frac{V_B}{R} = \sqrt{\frac{4g(H-h)}{3R^2}}}$$

Problem continued on the following page.

(b) (6) What is the horizontal distance the disk will travel before reaching its maximum height?

$$V_y = V_{oy} + at = V_{oy} - gt$$

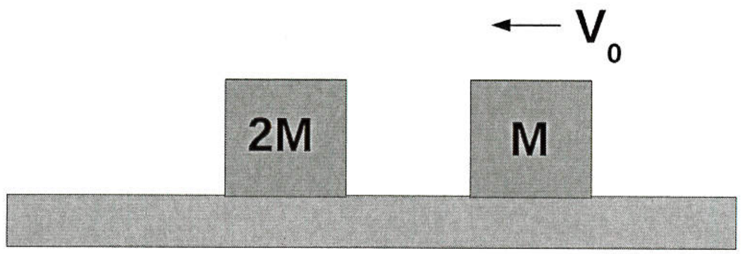
AT MAXIMUM HEIGHT, $V_y = 0$, $t = \frac{V_{oy}}{g} = \frac{V_B \sin 30^\circ}{g} = \frac{V_B}{2g}$

$$X - X_0 = V_{ox} t = V_B \cos 30^\circ \times \frac{V_B}{2g}$$

$$= \sqrt{\tfrac{3}{4}}\, \frac{V_B^2}{2g} = \sqrt{\tfrac{3}{4}} \cdot \frac{4g(H-h)}{3} \cdot \frac{1}{2g}$$

$$\boxed{X - X_0 = \sqrt{\tfrac{1}{3}}\,(H-h)}$$

8. (16 pts) Two blocks, as shown in the figure, are on a frictionless surface. A block with mass M slides to the left with speed v_0, until it collides elastically with a block of mass $2M$ which is initially at rest. Calculate the final velocities (magnitude and direction) of the two blocks, $\vec{v_1}$ and $\vec{v_2}$.



Conserve Energy (Elastic)

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

$$v_0^2 = v_1^2 + 2v_2^2$$

$$v_1 = \sqrt{v_0^2 - 2v_2^2}$$

Conserve Momentum (No extl Forces)

$$mv_0 = mv_1 + 2mv_2$$

$$v_0 = v_1 + 2v_2$$

$$v_0 = \sqrt{v_0^2 - 2v_2^2} + 2v_2$$

Now some arithmetic...

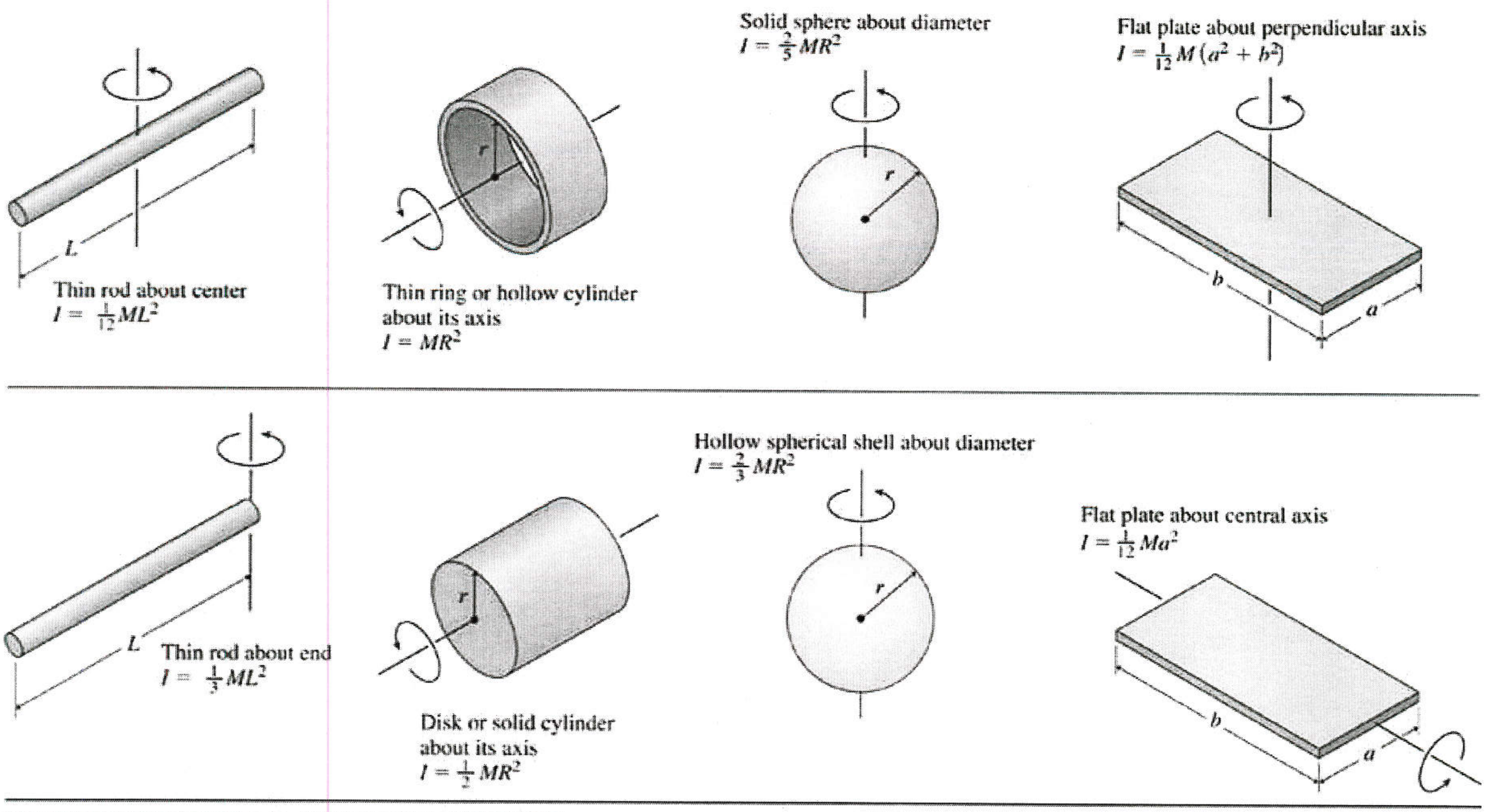
$$(v_0 - 2v_2)^2 = v_0^2 - 2v_2^2$$

$$\cancel{v_0^2} + 4v_2^2 - 4v_0v_2 = \cancel{v_0^2} - 2v_2^2$$

$$\boxed{\vec{v_2} = +\frac{2}{3}v_0 \text{ ; To Left}}$$

$$v_1 = v_0 - 2v_2 = v_0 - \frac{4}{3}v_0 = -\frac{1}{3}v_0$$

$$\boxed{\vec{v_1} = \frac{1}{3}v_0 \text{ ; To Right}}$$


Thin rod about center
$I = \frac{1}{12}ML^2$
L
Thin ring or hollow cylinder about its axis
$I = MR^2$
r
Solid sphere about diameter
$I = \frac{2}{5}MR^2$
r
Flat plate about perpendicular axis
$I = \frac{1}{12}M(a^2 + b^2)$
b
a
Thin rod about end
$I = \frac{1}{3}ML^2$
L
Disk or solid cylinder about its axis
$I = \frac{1}{2}MR^2$
r
Hollow spherical shell about diameter
$I = \frac{2}{3}MR^2$
r
Flat plate about central axis
$I = \frac{1}{12}Ma^2$
b
a