Physics 3210, Spring 2019

Exam #1

Name: Solutions

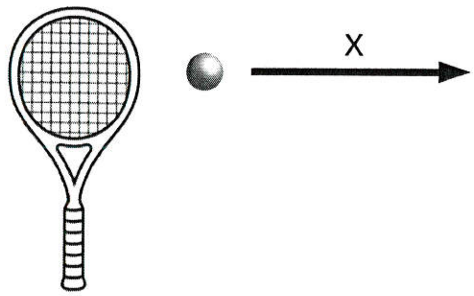
Signature:

UID:

Please read the following before continuing:

- Show all work in answering the following questions. Partial credit may be given for problems involving calculations.
- Be sure that your final answer is clearly indicated, for example by drawing a box around it.
- Be sure that your cellphone is turned off.
- Your signature above indicates that you have neither given nor received unauthorized assistance on any part of this exam.
- Thanks, and good luck!

1. (6 pts) A tennis racket strikes a tennis ball (mass = 58.5 g) which is initially at rest. The racket exerts a constant force on the ball for 0.1 seconds, after which the ball has a speed of 45 meters/second in the positive x-direction. Neglect air resistance.

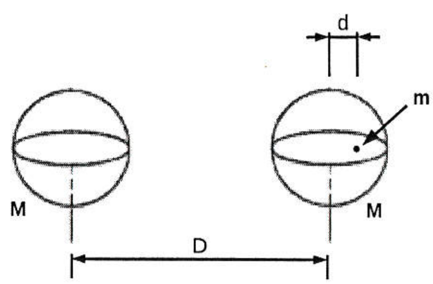


What was the force exerted **by the ball on the racket**? Give both magnitude and direction.

$$\vec{F}_{BALL,RACKET} = m\frac{\Delta V}{\Delta t}\hat{x} = \frac{(0.0585\ kg)(45\ m/s)}{0.1\ s}\hat{x} = 26.325\,N\,\hat{x}$$

$$\vec{F}_{RACKET,BALL} = -\vec{F}_{BALL,RACKET} = \boxed{-26.325\,N\,\hat{x}\ \text{ i.e. to THE LEFT}}$$

2. (6 pts) Two spherical shells of mass M are placed a distance D apart, as shown in the figure. One shell contains a pellet of mass m a distance d from the center of the shell, as shown.

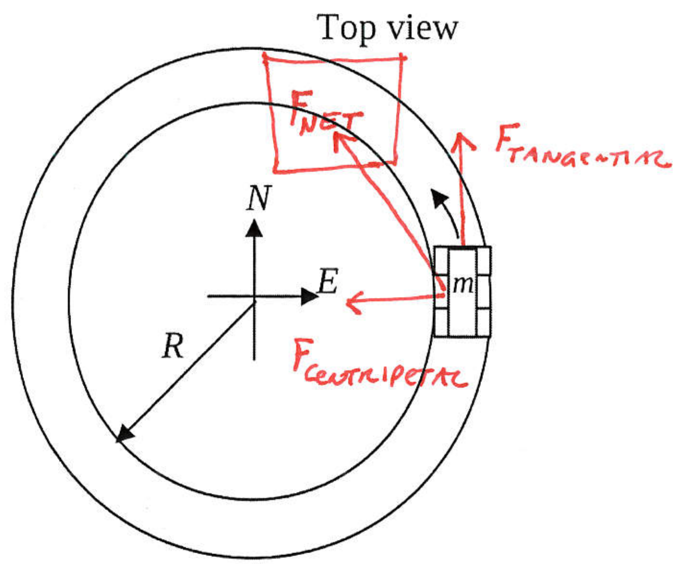


Find the net gravitational force (magnitude and direction) on the pellet.

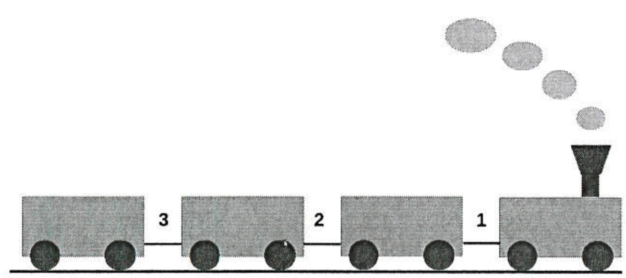
THE SHELL ON THE RIGHT EXERTS NO FORCE ON THE PELLET.
THE SHELL ON THE LEFT EXERTS SAME FORCE AS A POINT MASS AT ITS CENTER.

$$\boxed{\vec{F} = \frac{GMm}{(D+d)^2},\ \text{TO LEFT}}$$

3. (6 pts) A car is driving on a circular track as shown in the figure below. At the time shown, the driver hits the gas so that the car's tangential speed is increasing. On the figure below, sketch the direction of the net frictional force being applied by the track on the car. Assume the wheels are rolling without slipping.



4. (6 pts) A locomotive engine is pulling three boxcars behind it, each having equal mass M. The cars are connected by cables numbered 1, 2 and 3. The train has acceleration $\vec{a}$ to the right. Determine the tension in each of the three cables. Ignore friction.

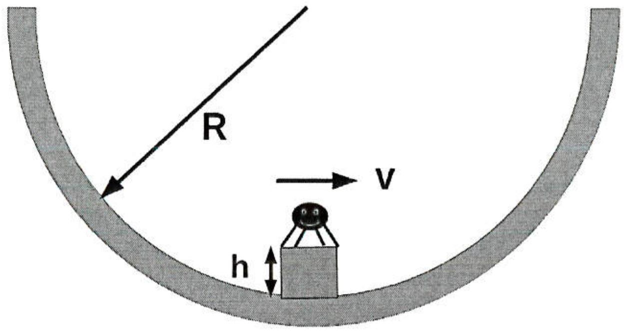


ALL CARS EXPERIENCE THE SAME ACCELERATION $\vec{a}$

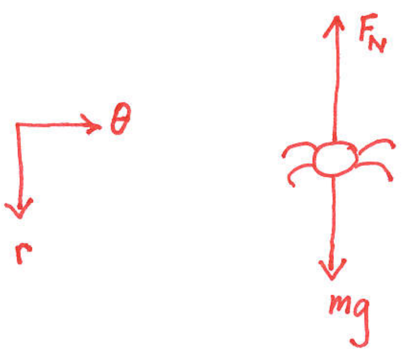
TENSION IN CABLES 1, 2, 3 MUST ACCELERATE 3m, 2m, 1m respectively.

$$\boxed{\begin{aligned} F_{T_1} &= 3ma \\ F_{T_2} &= 2ma \\ F_{T_3} &= ma \end{aligned}}$$

5. (12 pts) A spider is clinging to an ice cube of height h, which is sliding without friction in a circular bowl of radius R. At the bottom of the bowl, the normal force of the ice cube on the spider is equal to three times the spider's weight.



(a) (4) At the instant shown, draw a free-body diagram of the spider indicating all forces acting on it.



(AT THE INSTANT SHOWN, THE SPIDER IS EXPERIENCING NO HORIZONTAL ACCELERATION)

(b) (8) Calculate the speed v of the spider at the bottom of the bowl.

$$\Sigma F_r = mg - F_N = m a_r = -\frac{mv^2}{(R-h)}$$

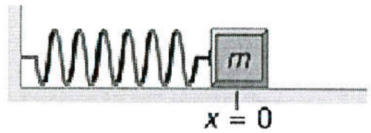
$$F_N = 3mg$$

$$mg - 3mg = \frac{-mv^2}{(R-h)}$$

$$2g = \frac{v^2}{(R-h)}$$

$$\boxed{V = \sqrt{2g(R-h)}}$$

6. (16 pts) A *simple harmonic oscillator* consists of a block of mass $m = 1$ kg sliding on a frictionless surface, under the influence of a massless spring. The force exerted by the spring on the block is given by $\vec{F} = -kx\hat{x}$, where x is the coordinate of the block relative to the equilibrium position $x = 0$, and $k = 9.87$ Newtons/meter. x is positive to the right.



(a) (4) What is the period of oscillation of the block?

$\omega = \sqrt{\frac{k}{m}}$; $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0 kg}{9.87 N/m}} = \boxed{2.00 sec}$

(b) (6) A general solution for the motion of the block is given by

$$x(t) = A\cos(\omega t + \phi)$$

where A and ϕ are constants of integration. If the block is released at rest from $x = -0.4$ meters at $t = 0$, find A and ϕ.

$x(t) = A\cos(\omega t + \phi)$ $\qquad$ $v(t) = \dot{x}(t) = -\omega A \sin(\omega t + \phi)$

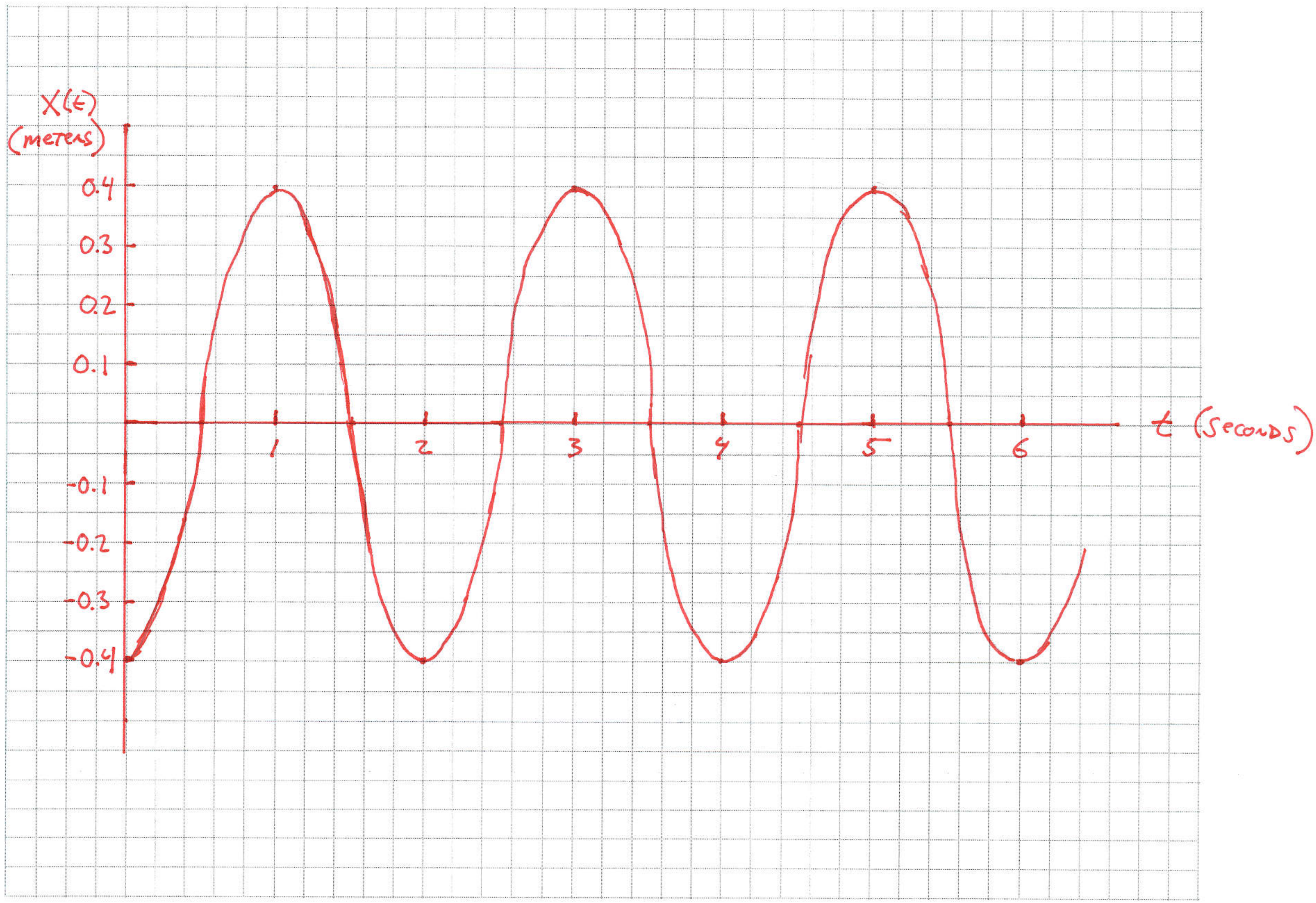
$x(0) = A\cos(\phi) = -0.4m$ $\qquad$ $v(0) = -\omega A \sin(\phi) = 0$

So EITHER $\boxed{A = -0.4m, \ \phi = 0}$

OR

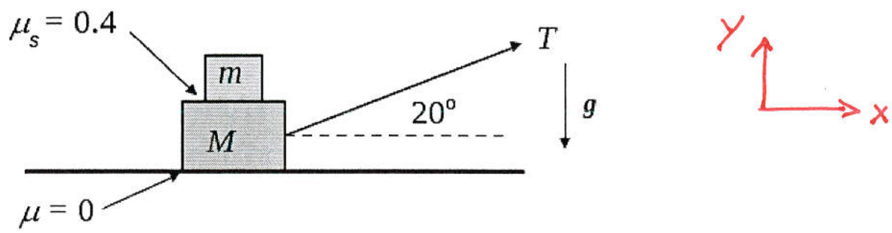
$\boxed{A = +0.4m, \ \phi = \pi}$ WILL WORK

(c) (6) Sketch the motion for several cycles on the grid on the following page.



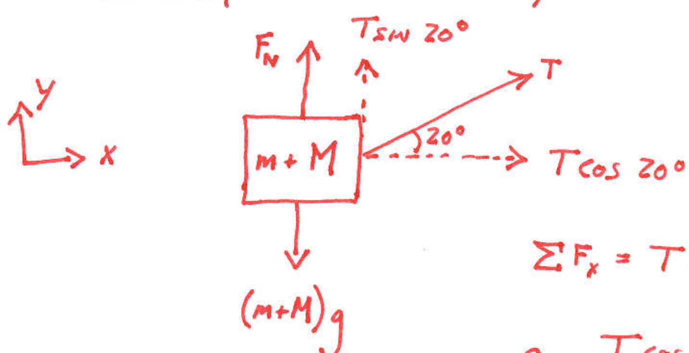
X(t)
(METERS)
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
t (SECONDS)
1
2
3
4
5
6

7. (12 pts) A block of mass $m = 0.6$ kg rests on top of a block of mass $M = 1.8$ kg. A string attached to the block of mass M is pulled so that its tension is $T = 8.0$ N at a 20° angle to the horizontal as shown. The blocks move together. The coefficient of static friction at the surface between the blocks is $\mu_s = 0.4$; there is no friction at the surface between block M and the floor.



(a) (4) What is the acceleration of the two-block system?

WHILE THEY MOVE TOGETHER, EFFECTIVE FBD IS:



$$\Sigma F_x = T\cos 20° = (m+M)a$$

$$a = \frac{T\cos 20°}{m+M} = \frac{(8.0\text{ N})\cos 20°}{(0.6+1.8)\text{kg}} = \boxed{3.13\text{ m/s}^2}$$

(b) (2) What is the *direction* of the frictional force being exerted on the top block m?

TO THE $\boxed{\text{RIGHT.}}$

(c) (6) The tension T is now increased. What is the maximum tension T_{max} with which the string can be pulled such that the blocks continue to move together (*i.e.* that the block of mass m does not start to slide on top of the block of mass M)?

FORCE OF STATIC FRICTION $F_S = \mu_S F_N = \mu_S mg$

WHEN $F_S < ma$, m WILL START TO SLIP.

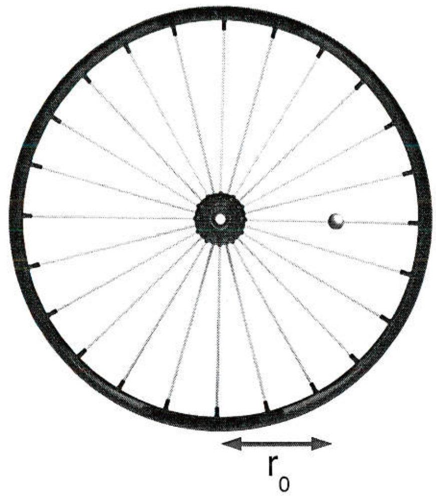
$$\mu_S \cancel{m} g < \cancel{m}\left(\frac{T\cos\theta}{m+M}\right)$$

$$T_{MAX} = \frac{\mu_S (m+M) g}{\cos\theta}$$

$$= \frac{0.4\,(0.6\text{ kg} + 1.8\text{ kg})\,9.81\text{ m/s}^2}{\cos 20^\circ}$$

$$= \boxed{10.02 \text{ NEWTONS}}$$

8. (20 pts) A bead can slide without friction on the spoke of a wheel. The wheel spins counterclockwise at a rate of 2.0 radians per second. At $t = 0$, the bead is a distance $r(0) = r_0 = 15$ cm from the center of the wheel, with zero radial speed.



(a) (6) How many revolutions per minute does the wheel complete?

$$\frac{2 \text{ radians}}{\text{sec}} \times \frac{1 \text{ revolution}}{2\pi \text{ radians}} \times \frac{60 \text{ sec}}{\text{minute}} = \boxed{\frac{120}{2\pi} \text{ rev/min}}$$

$$= \boxed{19.1 \text{ rev/min}}$$

(b) (6) What is the speed (in meters/second) of the bead at $t = 0$?

$$V_0 = r_0 \omega = (0.15 \text{ m})(2.0 \text{ rad/sec}) = \boxed{0.3 \text{ m/sec}}$$

(c) (8) What is $\ddot{r} = d^2r/dt^2$ of the bead at $t = 0$? Is $\ddot{r}$ constant, increasing with time or decreasing with time?

ACCELERATION IN POLAR COORDINATES IS GIVEN BY:

$$\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{r} + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{\theta}$$

No force acts in the $\hat{r}$ direction, so

$$\ddot{r} - r\dot{\theta}^2 = 0$$

$$\ddot{r} = r\dot{\theta}^2 = r\omega^2$$

$$\ddot{r}(t=0) = r_o\omega^2 = 0.15\,m\left(\frac{2\,rad}{sec}\right)^2 = \boxed{0.6\,m/s^2}$$

$\boxed{\ddot{r} \text{ INCREASES WITH TIME}}$

(d) FOR FUN ONLY: If the rim of the wheel is at $2r_o = 30$ cm, when will the bead reach the rim?

DIFF. EQ. $\ddot{r} = r\omega^2$ HAS GENERAL SOLUTION $r = Ae^{\omega t} + Be^{-\omega t}$

$$r(t=0) = A + B = r_o$$

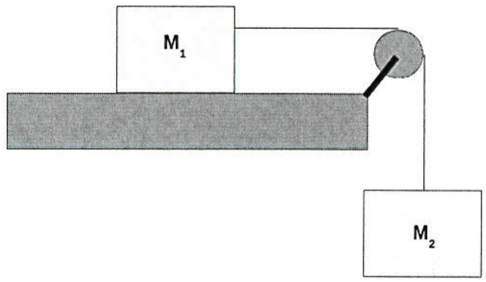
$$\dot{r}(t=0) = \omega A - \omega B = 0 \longrightarrow A = B = r_o/2$$

SO, $$r = r_o\frac{(e^{\omega t} + e^{-\omega t})}{2} = r_o \cosh \omega t$$

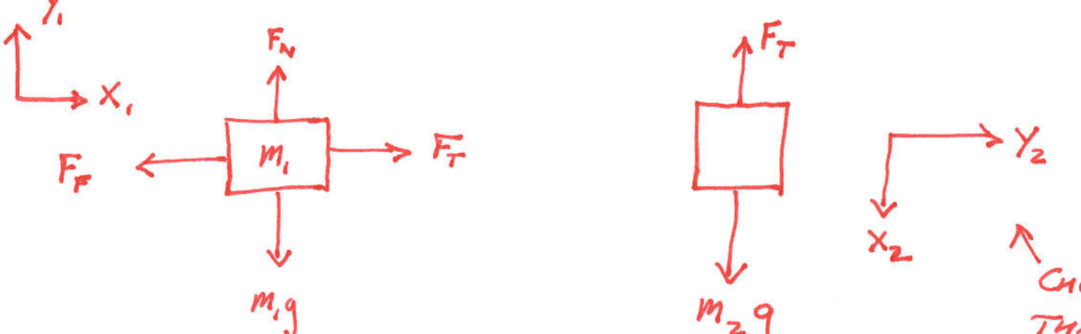
FIND TIME AT WHICH $r_o \cosh \omega t = 2r_o$

$$t = \frac{\operatorname{acosh}(2)}{2} = \boxed{0.658\,sec}$$

9. (16 pts) Two masses are connected as shown in the figure below. M_1 slides to the right on a horizontal table, with coefficient of kinetic friction μ_k. M_1 is connected to M_2 via a massless, frictionless pulley and a massless string.



(a) (8) Draw free body diagrams for masses M_1 and M_2. Indicate the coordinate system(s).



CHOOSE THIS SO THAT $X_1 = X_2$!

(b) (8) Derive a simple expression for the acceleration of M_1 in terms of M_1, M_2, μ_k and g. (If you're having trouble incorporating friction, do the calculation for a frictionless table for partial credit.)

(i) $\sum F_{x_1} = F_T - F_f = m_1 a$

$F_f = \mu_k m_1 g$ (iii)

(ii) $\sum F_{x_2} = m_2 g - F_T = m_2 a$

ADD (i) & (ii) $m_2 g - F_f = (m_1 + m_2) a$

SUBSTITUTE (iii) $m_2 g - \mu_k m_1 g = (m_1 + m_2) a$

$$\boxed{a = \frac{(m_2 - \mu_k m_1)}{(m_1 + m_2)} g}$$

{ No friction,

$a = \left(\frac{m_2}{m_1 + m_2}\right) g$ }