

Physics 3210, Spring 2018  
Exam #1

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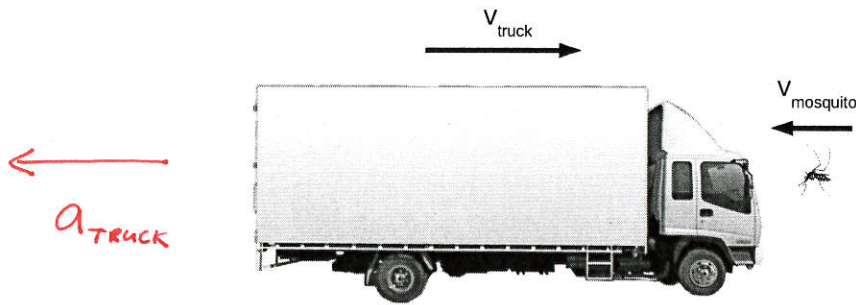
SOLUTIONS  
EB

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Please read the following before continuing:

- Show all work in answering the following questions. Partial credit may be given for problems involving calculations.
- Be sure that your final answer is clearly indicated, for example by drawing a box around it.
- Be sure that your cellphone is turned off.
- Your signature above indicates that you have neither given nor received unauthorized assistance on any part of this exam.
- Thanks, and good luck!

1. (4 pts) A large truck speeding down the highway strikes a mosquito.



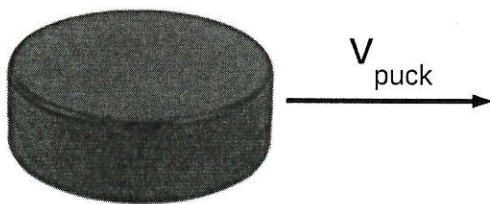
- (a) (2) Does the truck or the mosquito experience the force of greater magnitude during the collision? Explain.

By Newton's 3<sup>rd</sup> Law,  $\vec{F}_{\text{TRUCK, MOSQUITO}} = -\vec{F}_{\text{MOSQUITO, TRUCK}}$ .

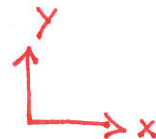
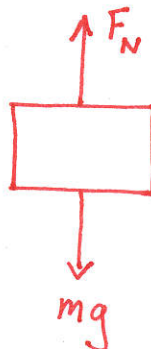
Magnitude of Force is THE SAME

- (b) (2) Indicate with an arrow on the figure the direction of the truck's *acceleration* during the collision.

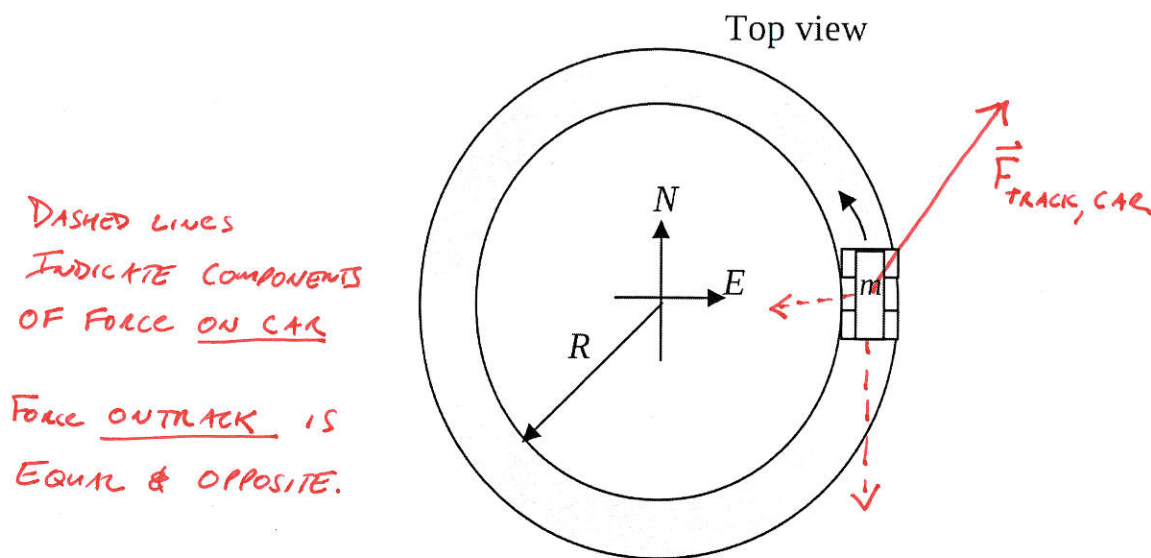
2. (4 pts) A hockey puck is sliding on wet ice with constant velocity to the right. Assuming there is no friction, draw a complete free body diagram for the hockey puck indicating all forces.



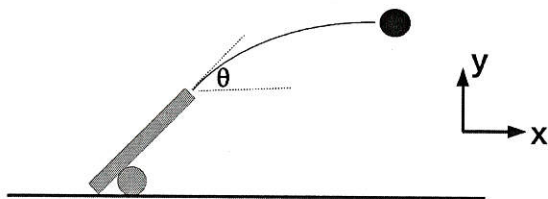
No Acceleration,  
 $\therefore$  No Force in x-Direction.



3. (4 pts) A car is driving on a circular track as shown in the figure below. At the time shown, the driver steps on brakes so that the car's tangential speed is decreasing. On the figure below, sketch the direction of the net frictional force being applied by the car's wheels on the track. The wheels are rolling without slipping.



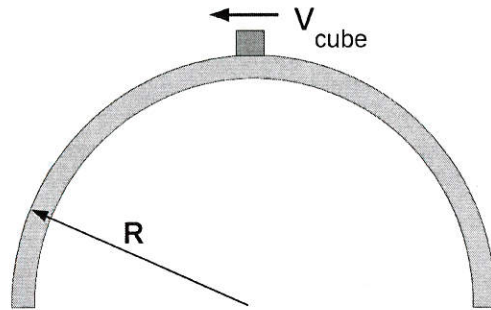
4. (16 pts) A cannon fires a projectile with initial speed  $v$  at an angle of  $\theta$  with respect to horizontal. The projectile flies under the influence of uniform gravity, before returning to Earth. Assume air resistance is negligible.



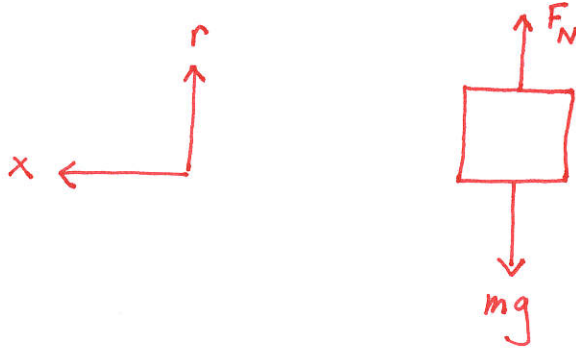
Indicate with a "T" or "F" whether the following statements are true or false while the projectile is in flight:

- (a) F The  $y$  component of the projectile's velocity is never zero.
- (b) T The horizontal ( $x$ ) component of the projectile's velocity remains constant.
- (c) T The acceleration of the projectile remains constant during its upward flight.
- (d) F The vertical ( $y$ ) component of the projectile's velocity remains constant.
- (e) T The horizontal acceleration of the projectile is zero.
- (f) F The vertical acceleration of the projectile is zero.
- (g) T The  $x$  component of the projectile's velocity is never zero.
- (h) F The minimum speed of the projectile during its flight is equal to  $v \sin \theta$ .

5. (10 pts) An ice cube of mass  $m$  is sliding without friction on an inverted spherical bowl of radius  $R$ . Calculate the maximum value of the cube's speed  $v_{\text{cube}}$  at the instant shown, such that the cube does not fly off the bowl. Give your answer in terms of the variables  $m$ ,  $R$ , and the acceleration due to gravity  $g$ . *Hint: What is the value of the normal force just before the cube leaves the bowl?*



FBD



$$\sum F_r = F_N - mg = ma_r = -\frac{mv^2}{R}$$

JUST BEFORE FLYING OFF BOWL, NORMAL FORCE = 0 (HINT)

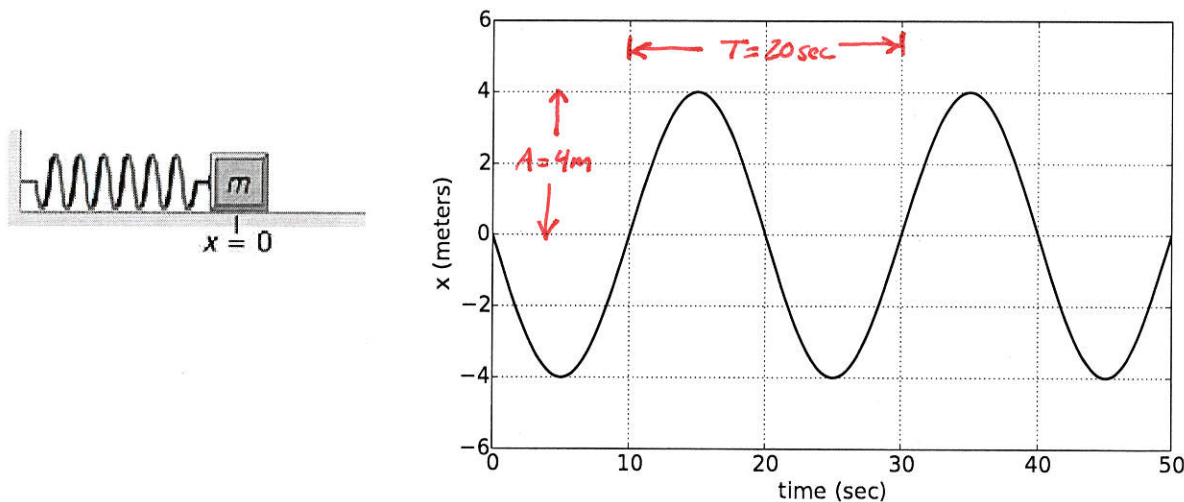
$$-mg = -\frac{mv^2}{R}$$

$$v_{\text{cube}} \leq \sqrt{gR}$$

6. (18 pts) A *simple harmonic oscillator* consists of a block of mass  $m$  sliding on a frictionless surface, under the influence of a massless spring. The force exerted by the spring on the block is given by

$$\vec{F} = -kx \hat{x}$$

where  $x$  is the coordinate of the block relative to the equilibrium position  $x = 0$ . A graph of the block's position versus time is shown below.



Answer the following questions.

- (a) (2) What is the period of oscillation of the block?

$T = 20 \text{ Sec}$  READ OFF THE GRAPH

- (b) (3) If  $m = 1.2 \text{ kg}$ , what is the value of the spring constant  $k$ ?

1<sup>ST</sup>,  $\omega = \frac{2\pi}{T} = \frac{2\pi}{20\text{sec}} = \frac{\pi}{10} \text{ s}^{-1}$ . 2<sup>ND</sup>  $\omega = \sqrt{\frac{k}{m}}$  so  $k = \omega^2 m = \left(\frac{\pi}{10\text{s}}\right)^2 \cdot 1.2\text{kg}$

- (c) (2) What is the block's position at  $t = 0 \text{ sec}$ ?

$x(t=0) = 0$  READ OFF GRAPH

$k = 0.118 \text{ N/m}$

- (d) (2) What is the first time at which the block's speed is zero?

FIRST TIME SLOPE =  $\frac{dx}{dt} = 0$  IS AT  $t = 5 \text{ sec}$

Problem continued on the following page.

(e) (3) What is the block's acceleration at  $t = 60$  sec? (I know ... its not on the graph!)

$$X(t) = A \cos(\omega t + \phi)$$

Curve crosses 0 every 10 sec.

$$V(t) = \dot{X}(t) = -\omega A \sin(\omega t + \phi)$$

$$\therefore X(60 \text{ sec}) = 0$$

$$a(t) = \ddot{X}(t) = -\omega^2 A \cos(\omega t + \phi) \\ = -\omega^2 X(t)$$

$$\therefore \boxed{a(60 \text{ sec}) = 0}$$

(f) (6) A general solution for the motion of the block is given by

$$x(t) = A \cos(\omega t + \phi)$$

where  $A$  and  $\phi$  are constants of integration. Determine  $A$  and  $\phi$  for the particular motion shown in the graph.

$$\boxed{A = 4 \text{ meters}} \quad \text{READ OFF GRAPH.}$$

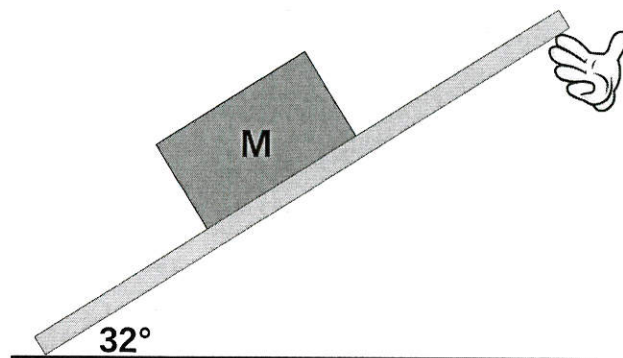
Need  $x$  or  $v$  AT SOME TIME TO DETERMINE  $\phi$

$$\text{e.g. } x(5s) = 4 \cos\left(\frac{\pi}{10} \cdot 5 + \phi\right) = -4$$

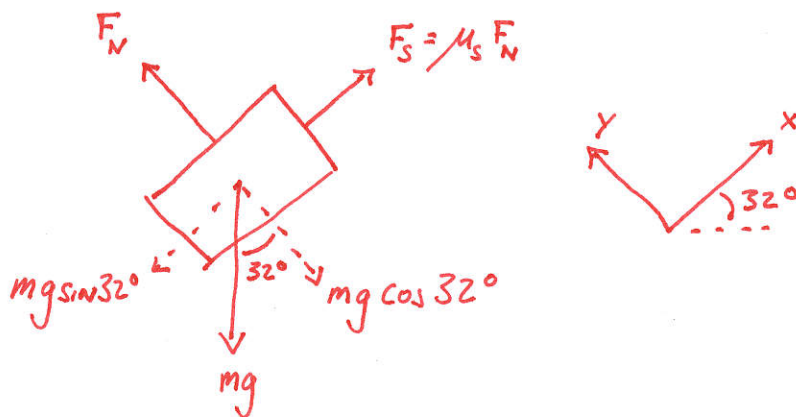
$$\therefore \boxed{\phi = \pi/2}$$



7. (12 pts) A block of mass  $m = 0.75 \text{ kg}$  sits at rest on a smooth (but not frictionless) surface. The ramp is slowly tilted by hand until it makes an angle of  $32^\circ$  with respect to horizontal, at which point the block begins to slide.



- (a) (6) Draw a *free body diagram* of the block just before it starts to slide.



- (b) (6) Calculate the *coefficient of static friction*  $\mu_s$  between the block and the ramp.

$$\sum F_x = \mu_s F_N - mg \sin 32^\circ = 0$$

$$\sum F_y = F_N - mg \cos 32^\circ = 0$$

$$F_N = mg \cos 32^\circ$$

$$\mu_s = \frac{mg \sin 32^\circ}{F_N} = \frac{mg \sin 32^\circ}{mg \cos 32^\circ} = \tan 32^\circ$$

$$\mu_s = 0.625$$

8. (16 pts) An satellite orbits the Earth at a constant distance  $R_o$  from the center of the Earth, at a constant speed  $v$ .

Useful Constants:

$$G = 6.673 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$$

$$M_E = 5.972 \times 10^{24} \text{ kg}$$

$$R_E = 6.371 \times 10^6 \text{ m}$$

- (a) (4) Write down the general expression for the velocity vector  $\vec{v}$  and acceleration vector  $\vec{a}$  in polar coordinates.

$$\vec{V} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}$$

$$\vec{a} = [\ddot{r} - r \dot{\theta}^2] \hat{r} + [r \ddot{\theta} + 2\dot{r} \dot{\theta}] \hat{\theta}$$

- (b) (6) Simplify the expressions for  $\vec{v}$  and  $\vec{a}$ , for the satellite's uniform circular motion.

Uniform motion,  $\dot{\theta} = \omega = \text{CONSTANT}$   
 $r = R_o = \text{CONSTANT}$

$$\vec{V} = 0 \hat{r} + R_o \omega \hat{\theta} = \underline{\underline{R_o \omega \hat{\theta}}}$$

$$\vec{a} = [0 - R_o \omega^2] \hat{r} + [0 + 0] \hat{\theta} = \underline{\underline{-R_o \omega^2 \hat{r}}}$$

Problem continued on the following page.



- (c) (6) Use the above to evaluate the velocity of the satellite as a function of  $G$ ,  $M_E$ , and  $R_o$ . Derive a numerical answer (in kilometers/second) for the case  $R_o = 2R_E$ .

$$F = ma$$

$$\frac{G M_E \cancel{m_{SAT}}}{R_o^2} = \frac{\cancel{m_{SAT}} V^2}{R_o} \quad \text{using } V = \omega R$$

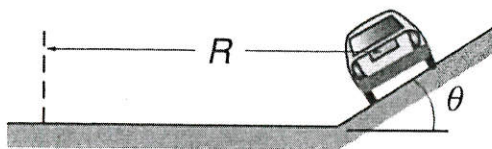
$$V = \sqrt{\frac{G M_E}{R_o}} = \sqrt{\frac{G M_E}{2 R_E}}$$

$$= \sqrt{\frac{(6.673 \times 10^{-11}) (5.972 \times 10^{24})}{2 \times 6.371 \times 10^6}}$$

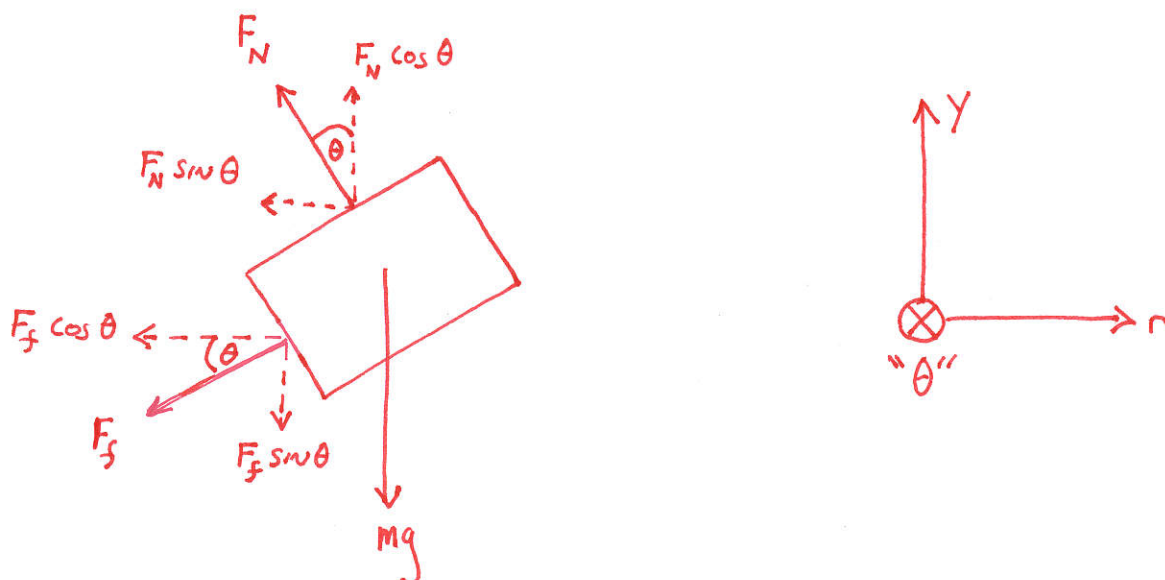
$$= 5,592 \text{ m/s}$$

$$V = 5.592 \text{ km/sec}$$

9. (16 pts) A particular turn on a highway has radius is  $R = 100$  m. The road is banked at an angle  $\theta$ , and the coefficient of friction between the wheels of the car and the road is  $\mu_s$ .



- (a) (6) Draw a complete *free body diagram* of a car on this curve, assuming the car is just about to start sliding off the road to the right.



*Problem continued on the following page.*

AND  $R = 100 \text{ m}$

(b) (10) If  $\mu_s = 0.10$  and  $\theta = 20^\circ$ , what should the speed limit be so that cars will not slide off the road?

$$(i) \quad \sum F_r = -F_f \cos \theta - F_N \sin \theta = -\frac{mv^2}{R}$$

$$(ii) \quad \sum F_y = F_N \cos \theta - F_f \sin \theta - mg = 0$$

Also  $(iii) \quad F_f = \mu_s F_N$

Now, ALGEBRA ...

$$\mu_s F_N \cos \theta + F_N \sin \theta = \frac{mv^2}{R}$$

$$F_N \cos \theta - \mu_s F_N \sin \theta = mg$$

Divide

$$\frac{\mu_s \cos \theta + \sin \theta}{\cos \theta - \mu_s \sin \theta} = \frac{v^2}{Rg}$$

$$V = \sqrt{Rg \left( \frac{\mu_s \cos \theta + \sin \theta}{\cos \theta - \mu_s \sin \theta} \right)}$$

Plug-in numbers

$$V = 21.7 \text{ m/sec}$$