Physics 3210 Spring 2019 Discussion #23 Answers

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1

This first question asks us to find an expression for the lifetime of the muon as seen by the Earth. Let's begin by considering the space-time interval of the muon event for the muon frame and the Earth frame. In the muon reference frame the muon does not move and the muon has a lifetime of τ. Thus the following is the spacetime interval for the muon in the muon frame:

$$\Delta s^2 = c^2\tau^2 \tag{1}$$

In the Earth reference frame the muon lasts some time t and travels some distance. Thus the spacetime interval for the Earth reference frame is the following:

$$\Delta s^2 = c^2t^2 - \Delta x^2 \tag{2}$$

We know the speed that the muon travels at with respect to Earth, thus we may change (2) to the following:

$$\begin{aligned} \Delta s^2 &= c^2t^2 - (vt)^2 \\ &= (c^2 - v^2)t^2 \end{aligned} \tag{3}$$

Now let's set (1) and (3) equal to each other and solve for t. Doing so gives the following:

$$t = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

Note that we have just derived time dilation using the space-time interval.

2

For this part we are asked what the lifetime of the muon is as seen by Earth and the distance that the muon travels as seen by Earth. To find the lifetime all we have to do is plug in numbers into (4):

$$\begin{aligned} t &= \frac{2.2 \times 10^{-6}}{\sqrt{1 - 0.9998^2}} \\ &= 1.1 \times 10^{-4} \end{aligned} \tag{5}$$

To find the distance the muon travels as seen by Earth all we need is kinematics:

$$\begin{aligned} d_{earth} &= vt \\ &= 0.9998(3 \times 10^8 \frac{m}{s})(1.1 \times 10^{-4} s) \\ &= 32.995 km \end{aligned} \tag{6}$$

Note that this distance is more than enough to make it to the surface of the Earth. So special relativity helps us successfully explain why we can measure muons on the surface of the Earth.

3

This last section first asks us to determine the distance travelled by the muon as seen by the muon. In the muon's frame it sees the Earth travelling toward it with speed $0.9998c$. Thus the distance travelled as seen by the muon is the following:

$$\begin{aligned} d_{muon} &= v\tau \\ &= 0.9998(c \times 10^8 \frac{m}{s})(2.2 \times 10^{-6} s) \\ &= 659.898 m \end{aligned} \tag{7}$$

The final question asked is what is the ratio of the muon distance to the earth distance. Taking (7) and dividing that by (6) gives the following ratio:

$$\frac{d_{muon}}{d_{earth}} = 0.02$$

So the distance the muon travels as seen by the Earth is much greater than the distance the muon travels in it's reference frame.