1. *Muons* are elementary particles that are produced high in the Earth's atmosphere, at altitudes of 10 km or more, by cosmic rays. The average lifetime τ of a muon is about 2.2×10^{-6} seconds or 2.2 microseconds, when viewed in its rest frame. Given that, one might expect from classical kinematics that the distance d that muons can travel is about

$$\begin{aligned} d &\approx c\tau \\ &\approx (3 \times 10^8 \text{m/s})(2.2 \times 10^{-6} \text{ s}) \\ &\approx 660 \text{ meters} \end{aligned}$$

So why is it that a substantial number of muons are still detected at the Earth's surface?

(a) Starting with the *invariant spacetime interval*:

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2$$

derive an expression for the lifetime of the muon in the frame of reference of the Earth, if the muon's speed with respect to Earth is v. This is the *relativistic time dilation*.

(b) If $v = 0.9998\ c$, what muon lifetime will an observer at rest with respect to the Earth measure? What distance d_{earth} will the muon travel in that time?

(c) What distance d_{muon} does the *muon* think it has traveled with respect to the Earth during its lifetime? From the muon's perspective, the Earth is moving past it at $v = 0.9998\ c$. What is the ratio d_{muon}/d_{earth}?