1. A damped harmonic oscillator consists of a mass m sliding without friction on a horizontal surface, under the influence of a spring with constant k and a velocity-dependent drag force $\vec{F}_d = -b\vec{v}$. In class we derived the equation for the damped oscillator

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = 0$$

where x is the coordinate of the mass, $\gamma = b/2m$ and $\omega = \sqrt{k/m}$.

   (a) In the "critically damped" case, $\gamma = \omega$. Find a general solution, with two constants of integration, by assuming that $x(t)$ has the form

$$x(t) = u(t)e^{-\gamma t}$$

   (b) In the critically damped case, the damped oscillator starts at $x = x_0$ and is given an initial push towards the equilibrium position. Find the maximum initial speed v_0 of the oscillator such that it does not cross the equilibrium position.