Physics 3210 Spring 2019 Discussion #18 Answers

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1

First, let's focus on the magnitude of the centrifugal and centripetal force. Note that the position of Annie is perpendicular to the axis of rotation. From this we can get the following:

$$|F_{centrifugal}| = |F_{centripetal}| = m\omega r^2 \tag{1}$$

Note we are given a frequency, but we want an angular frequency. The following is the proper conversion for the frequency given:

$$\omega = 2\pi f = \frac{1}{2} rads/second \tag{2}$$

Now if we plug in all of the numbers we have, including (2), into (1) we get the following magnitude:

$$|F_{centrifugal}| = |F_{centripetal}| = 45N$$

Now let's focus on the directions of the two forces. We know that the centripetal force maintains circular motion, so the force must point towards the center of rotation. For Annie that means that the centripetal force must be in the $\hat{x}$ direction. The centrifugal force points in the opposite direction of the centripetal force, it it must point in the $-\hat{x}$ direction.

2

For this part we should note that there are no external torques acting on the system, so angular momentum should be conserved. From this we obtain the following equation:

$$I_i\omega_i = I_f\omega_f \tag{3}$$

Now we need to find the moments of inertia of the system while Annie is on the edge of the carousel and when we reaches the center of the carousel. First let's consider the moment of inertia of the system when Annie is at the edge of the carousel. We know that the moment of inertia of the system is the sum of the individual moments of inertia, so:

$$I_t ot = I_{carousel} + I_{Annie} + I_{Bob} \tag{4}$$

The carousel is a disk, and we will take Annie and Bob to be point sources for the sake of simplicity. With this in mind we get the following initial moment of inertia:

$$I_i = \frac{1}{2} m_c r^2 + m_A r^2 + m_B r^2 \tag{5}$$

When Annie reaches the center her moment of inertia is zero because she is a point source. So we obtain the following final moment of inertia:

$$I_f = \frac{1}{2} m_c r^2 + m_B r^2 \tag{6}$$

Solving (3) for I_f, plugging in (5) and (6) into (3), and then simplifying gives the following:

$$\omega_f = \frac{\frac{1}{2} m_c + m_A + m_B}{\frac{1}{2} m_c + m_B} \omega_i \tag{7}$$

Plugging in all of the masses and (2) gives the following final angular speed:

$$\omega_f = \frac{17}{28} radians/second \tag{8}$$

3

For this problem we only have to consider the direction of the Coriolis force on Annie. The Coriolis force is the following:

$$F_{Coriolis} = -2m \overrightarrow{\omega} \times \overrightarrow{v} \tag{9}$$

Ignoring all scalar quantities and plugging in the unit pointing directions of the vectors into (9) gives the following:

$$\begin{aligned} \hat{F}_{Coriolis} &= -(-\hat{z}) \times \hat{x} \\ &= \hat{z} \times \hat{x} \\ &= \hat{y} \end{aligned}$$

4

For this problem we will do the same thing as we did for the previous part. The azimuthal force is given by the following:

$$F_{Azimuthal} = -m\frac{d\omega}{dt} \times \hat{r} \tag{10}$$

Let's plug in the unit vectors into (10) for Bob:

$$F_{Azimuthal} = -mr\frac{d\omega}{dt}(-\hat{z}) \times \hat{x} \tag{11}$$

The magnitude of ω is increasing, so $\frac{d\omega}{dt}$ is positive. Thus we get the following direction after ignoring scalar values:

$$\begin{aligned}\hat{F}_{Azimuthal} &= \hat{z} \times \hat{x} \\ &= \hat{y}\end{aligned}$$