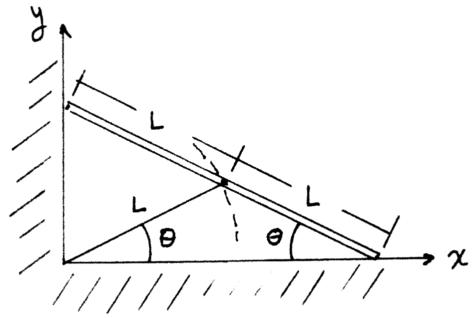
7.41 *Leaning plank*

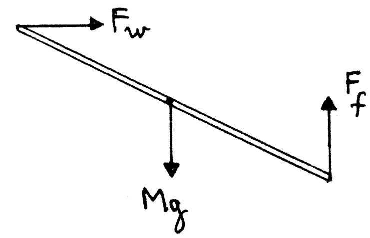


As long as the plank is in contact with the wall, the coordinates of its center of mass are

$$x = L\cos\theta \qquad y = L\sin\theta$$

$$x^2 + y^2 = L^2(\cos^2\theta + \sin^2\theta) = L^2$$

Until contact with the wall is lost, the center of mass moves on a circular path of radius L, as indicated in the upper sketch.
Because the wall and floor are frictionless, the force F_w exerted by the wall on the plank and the force F_f exerted by the floor are normal to the surfaces, as shown in the lower sketch.
The plank loses contact with the wall when $F_w = 0$, or, equivalently, when $\ddot{x} = F_w/M = 0$.



$$x = L\cos\theta \qquad \dot{x} = -L\sin\theta\,\dot{\theta}$$

$$\ddot{x} = -L\cos\theta\,\dot{\theta}^2 - L\sin\theta\,\ddot{\theta} = 0 \implies \dot{\theta}^2 = -\tan\theta\,\ddot{\theta} \quad (1)$$

There are no dissipative forces, so mechanical energy E is conserved. Let y_0 be the initial height of the center of mass above the floor.

$$E_i = Mgy_0 = E_f = Mgy + \frac{1}{2}M(L\dot{\theta})^2 + \frac{1}{2}I_0\,\dot{\theta}^2$$

$$Mgy_0 = MgL\sin\theta + \frac{1}{2}M(L\dot{\theta})^2 + \frac{1}{2}\left(\frac{1}{3}ML^2\right)\dot{\theta}^2 = MgL\sin\theta + \frac{2}{3}ML^2\,\dot{\theta}^2$$

$$y_0 = L\sin\theta + \frac{2}{3}\frac{L^2}{g}\dot{\theta}^2 \quad (2)$$

Differentiating Eq. (2),

$$0 = L\cos\theta\,\dot{\theta} + \frac{4}{3}\frac{L^2}{g}\dot{\theta}\,\ddot{\theta} \implies \ddot{\theta} = -\frac{3}{4}\frac{g}{L}\cos\theta$$

Using Eq. (1),

$$\dot{\theta}^2 = \frac{3}{4}\frac{g}{L}\sin\theta \quad (3)$$

Substituting Eq. (3) in Eq. (2),

$$y_0 = L\sin\theta + \frac{1}{2}L\sin\theta = \frac{3}{2}L\sin\theta = \frac{3}{2}y$$

so the plank loses contact with the wall at height

$$y = \frac{2}{3}y_0$$