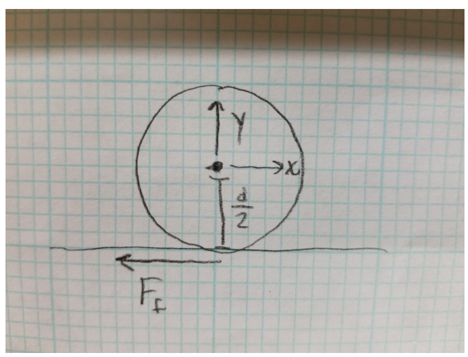
Physics 3210 Spring 2019 Discussion #17 Answers

Josh Peterson

March 6th 2019

1

For this problem we will use Newton's laws for angular and translational forces. The following is a force diagram for the bowling ball:



The force of friction from the interaction of the bowling ball and the bowling lane produces a translational force and a torque. Writing Newton's second law for both gives the following:

Linear:

$$F_f = ma \tag{1}$$

Angular:

$$-\frac{d}{2}\hat{y} \times F_f = I\alpha \tag{2}$$

Since the bowling ball is sliding we know that the force of friction is kinetic friction. So the friction force is the following:

$$F_f = -\mu_k mg\hat{x} \tag{3}$$

Plugging (3) into (1) and (2) gives the following expressions:

$$-\mu mg = ma \tag{4}$$

$$-\frac{d}{2}\hat{y} \times -\mu_k mg\hat{x} = -\frac{d}{2}\mu_k mg\hat{z} = I\alpha \tag{5}$$

The sign on (5) tells us that the rotation of the ball is clockwise. You can determine this fact with the right hand rule. We can immediately solve (4) for translational acceleration.

$$a = -\mu_k g \tag{6}$$

The bowling ball is a solid sphere, and for the sake of simplicity we can say that the ball has uniform density. Using the moment of inertia handout we have we may solve for the rotational acceleration.

$$\alpha = \frac{\frac{d}{2}\mu_k mg}{\frac{2}{5}m\left(\frac{d}{2}\right)^2} = \frac{5\mu_k g}{d} \quad \textit{clockwise} \tag{7}$$

Now we may integrate (6) and (7) to obtain translational speed and angular speed as a function of time.

$$v(t) = v_0 - \mu_k gt \tag{8}$$

$$\omega(t) = \frac{5\mu_k g}{d}t \tag{9}$$

Now we want to find the time at which the ball stops slipping. To do this we need to consider the no slip condition, which is given as the following:

$$v = \frac{d}{2}\omega \tag{10}$$

Plugging (10) into (9) gives the following expression:

$$v = \frac{5\mu_k g}{2}t \tag{11}$$

We can plug (11) into (8) to obtain the time the bowling ball stops slipping. Doing so and solving for t gives the following:

$$t = \frac{2v_0}{7\mu_k g} \tag{12}$$

The last thing the first part of the question asked for was the position at which the ball stops slipping. We have the time that the ball stops slipping, so all we have to do is find translational position as a function of time and plug (12) in. Integrating (8) gives us the translational position as a function of time.

$$x(t) = v_0 t - \frac{1}{2}\mu_k gt^2 \tag{13}$$

Plugging (12) into (13) gives the position at which the ball stops slipping.

$$x = \frac{12v_0^2}{49\mu_k g} \tag{14}$$

2

The first thing this section asks for is the speed the ball travels when it doesn't slip. We may easily obtain this by plugging the time given by (12) into the translational speed equation given by (8). This gives the following speed:

$$v = \frac{5}{7} v_0 \tag{15}$$

After this the question asks to compare the translational and rotation kinetic energies. First, let's find the translational kinetic energy. Plugging (15) into the kinetic energy equation gives the following:

$$T_t = \frac{25}{98} m v_0^2 \tag{16}$$

Now let's take (15), the no slip condition, and the moment of inertia of a solid sphere and plug them into the rotational kinetic energy equation. Careful manipulation reveals a direct comparison with the kinetic energy.

$$\begin{aligned} T_r &= \frac{1}{2} \left(\frac{2}{5} m \left(\frac{d}{2} \right)^2 \right) \left(\frac{2}{d} \frac{5}{7} v_0 \right)^2 \\ &= \frac{2}{5} \left(\frac{25}{98} m v_0^2 \right) \\ &= \frac{2}{5} T_t \end{aligned}$$