Physics 3210 Spring 2019 Discussion #15 Answers

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1

First we need to know the moment of inertia for a rod and sphere with the rotation axis about the center of mass. These are given to you in the handout as the following:

$$I_{rodCM} = \frac{1}{12}mL^2 \tag{1}$$

$$I_{sphereCM} = \frac{2}{5}MR^2 \tag{2}$$

Now we may use the parallel axis theorem to find the moments of inertia for the rod and the sphere with the rotation axis located at the end of the rod.

$$I_{rod} = \frac{1}{12}mL^2 + m(\frac{L}{2})^2 \tag{3}$$

$$I_{sphere} = \frac{2}{5}MR^2 + M(L+R)^2 \tag{4}$$

The total moment of inertia is then the moment of inertia of the rod plus the moment of inertia of the sphere.

$$\begin{aligned} I &= I_{rod} + I_{sphere} \\ &= \frac{1}{12}mL^2 + m(\frac{L}{2})^2 + \frac{2}{5}MR^2 + M(L+R)^2 \\ &= \frac{1}{3}mL^2 + \frac{7}{5}MR^2 + ML^2 + 2MLR \end{aligned}$$

2

We need to begin by finding the center of mass of the object. I will choose to set my coordinate system such that the origin is at the connection between the rod and the sphere and the rod lies along the x axis. If this is the case then the following is the center of mass:

$$X_{CM} = \frac{MR - \frac{mL}{2}}{M + m} \tag{5}$$

Now we want to set the origin of the coordinate system to the center of mass location. The distance between the centers of mass of the individual objects and the center of mass of the whole system is the following:

$$d_{rod} = \frac{L}{2} + X_{CM} \tag{6}$$

$$d_{sphere} = R - X_{CM} \tag{7}$$

Now we may use the parallel axis theorem with (6) and (7) to write the moment of inertia:

$$I = \frac{1}{12}mL^2 + m(\frac{L}{2} + X_{CM})^2 + \frac{2}{5}MR^2 + M(R - X_{CM})^2 \tag{8}$$

Plugging (5) into (8) and simplifying leads to the following moment of inertia:

$$I = \frac{1}{3}mL^2 + \frac{7}{5}MR^2 - (M + m)X_{CM}^2 \tag{9}$$

If you now use parallel axis theorem with this moment of inertia you can recover the answer in part 1, so we know that this is correct.

3

We will use parallel axis theorem on the sphere and the rod:

$$I_{rod} = \frac{1}{12}mL^2 + m(\frac{L}{2} + 2R)^2 + \tag{10}$$

$$I_{sphere} = \frac{2}{5}MR^2 + M(R)^2 \tag{11}$$

Adding (10) and (11) gives the moment of inertia of the rod and sphere with the axis of rotation at the end of the sphere.

$$\begin{aligned} I &= \frac{1}{12}mL^2 + m(\frac{L}{2} + 2R)^2 + \frac{2}{5}MR^2 + M(R)^2 \\ &= \frac{1}{3}mL^2 + \frac{7}{5}MR^2 + 4mR^2 + 2mLR \end{aligned}$$