1. A mass $m$ is free to slide one a frictionless table, and is connected by a string that passes through a hole in a table to a mass $M$ hanging below the table. See the figure below. Assume that $M$ can only move vertically, and that the string always remains taut.

(a) Use the Lagrangian method to find the equations of motion of the system, with $r$ and $\theta$ (see figure) as generalized coordinates.
(b) Show that the angular momentum $L=m r^{2} \dot{\theta}$ of mass $m$ is a constant of the motion. Find an expression for the radial acceleration $\ddot{r}$ for a particular $L$.
(c) There is a particular radius $r_{0}$, for a given $L$, such that $m$ will undergo uniform circular motion. What is this radius?
