Physics 3210 Spring 2019 Discussion #12 Answers

Josh Peterson

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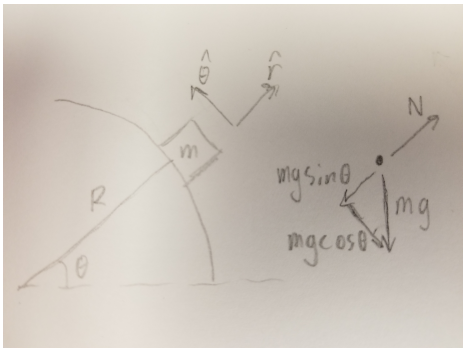
As long as the ice cube is on the bowl it is undergoing circular motion. Considering that, here is a force diagram of the ice cube:



Writing out Newton's second law for the $\hat{r}$ direction while the ice cube is on the bowl gives the following equation:

$$N - mg\sin\theta = -\frac{mv^2}{R} \tag{1}$$

Now let's consider when the ice cube leaves the bowl. If the ice cube loses contact with the bowl then the bowl cannot apply any force to the ice cube. Thus, the ice cube falls off the bowl when $N = 0$. Setting $N = 0$ in equation (1) and simplifying gives the following condition for when the ice cube falls off:

$$g\sin\theta = \frac{v^2}{R} \tag{2}$$

We see here that we need to find speed as a function of the ice cube angle. The easiest way to do this is with conservation of energy. The initial energy of the ice cube is just the potential energy of the ice cube sitting on the bowl:

$$E_i = mgR \tag{3}$$

As the ice cube slides down it gains kinetic energy and loses potential energy. So at some time after the ice cube starts to slide it has the following energy:

$$E_f = \frac{1}{2}mv^2 + mgR\sin\theta \tag{4}$$

By the conservation of energy we know $E_i = E_f$. This gives us the following:

$$mgR = \frac{1}{2}mv^2 + mgR\sin\theta \tag{5}$$

Solving (5) for v^2 gives us the following:

$$v^2 = 2gR(1 - \sin\theta) \tag{6}$$

Now we can use (6) and (2) to solve for the angle at which the block falls off of the bowl.

$$\begin{aligned} g\sin\theta &= 2g(1 - \sin\theta) \\ \implies \sin\theta &= \frac{2}{3} \end{aligned} \tag{7}$$

The reason we just solve for $\sin\theta$ instead of θ will become apparent really soon. The problem asks for the vertical distance from the top of the bowl that the ice cube falls off of the bowl. We can use trigonometry to find the vertical distance travelled by the block as a function of θ. Doing so gives the following:

$$x = R - R\sin\theta \tag{8}$$

Plugging (7) into (8) gives the following vertical distance where the ice cube falls off of the bowl:

$$\begin{aligned} x &= R\left(1 - \frac{2}{3}\right) \\ &= \frac{R}{3} \end{aligned}$$