Physics 3210 Spring 2019 Discussion #11 Answers

Josh Peterson

February 13th 2019

From lecture we know we can parameterize $\hat{r}$ with respect to x with the following equation:

$$\overrightarrow{r} = x\hat{x} + f(x)\hat{y} \tag{1}$$

With this we may find what $d\hat{s}$ is by taking differentials of (1):

$$d\overrightarrow{s} = dx\hat{x} + df(x)\hat{y} \tag{2}$$

where:

$$df(x) = \frac{d}{dx}(f(x))dx \tag{3}$$

In this problem we are given the following equation:

$$y^2 = x \tag{4}$$

The graph shows the solutions where y is positive, so we know that y is the following:

$$y = \sqrt{x} \tag{5}$$

Plugging (5) into (3) gives us the following differential:

$$dy = \frac{1}{2\sqrt{x}}dx \tag{6}$$

Now plugging (6) into (2) gives the following:

$$d\overrightarrow{s} = dx\hat{x} + \frac{1}{2\sqrt{x}}dx\hat{y} \tag{7}$$

Now we should rewrite $\overrightarrow{A}$ In terms of x. Plugging (5) into $\overrightarrow{A}$ gives us the following:

$$\overrightarrow{A} = x^2\hat{x} + x\hat{y} + z^2\hat{z} \tag{8}$$

Now we can find $\overrightarrow{A} \cdot d\overrightarrow{s}$:

$$\overrightarrow{A} \cdot d\overrightarrow{s} = x^2 dx + \frac{\sqrt{x}}{2} dx \tag{9}$$

Now let's integrate (9) over the interval $0 < x < 2$:

$$\begin{aligned} \int \overrightarrow{A} \cdot d\overrightarrow{s} &= \int_0^2 \left(x^2 + \frac{\sqrt{x}}{2} \right) dx \\ &= \left(\frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{3} \right) \Bigg|_0^2 \\ &= \frac{8 + 2\sqrt{2}}{3} \\ &\approx 3.609 \end{aligned}$$