Physics 3210 Spring 2019 Discussion #8 Answers

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This problem is fairly simple if we carefully consider Newton's second law for the center of mass of the rocket before and after the explosion. Before the explosion the following equations hold true for the rocket:

$$\hat{x}: \quad M\ddot{x} = 0 \tag{1}$$

$$\hat{y}: \quad M\ddot{y} = -Mg \tag{2}$$

Let's now consider the equations of motion after the rocket explodes. Let's take R to be the center of mass. Then using Newton's second law in the $\hat{x}$ direction gives us the following:

$$M\ddot{R}_x = F_x = 0 \tag{3}$$

This has the same exact form as (1). Let's now consider the $\hat{y}$ direction:

$$M\ddot{R}_y = -m_1 g - m_2 g = Mg \tag{4}$$

This equation has the same form as (2). We see from looking at Newton's second law that the equations of motion for the center of mass are the same before and after the explosion. Thus, the center of mass continues on the same trajectory as it had before the explosion.

This fact holds true until there is some external force other than gravity that acts on one or both of the pieces. So, if either piece lands before the other then we can no longer say the center of mass follows the normal trajectory. Fortunately, the problem states that the pieces fly apart horizontally. This little fact implies that the pieces will land at the same time, and so the center of mass will "land" when the pieces land.

Since the speed in the $\hat{x}$ direction is constant, and the rocket achieves maximum height at $x = L$, we know that the center of mass must reach the ground when $x = 2L$. With this information we can find where the second piece lands with the center of mass of the two pieces as they land.

$$
\begin{aligned}
\bar{X} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\
\implies 2L &= \frac{m(0) + 3m x_2}{m + 3m} \\
\implies 2L &= \frac{3x_2}{4} \\
\implies x_2 &= \frac{8L}{3}
\end{aligned}
$$