Physics 3210 Spring 2019 Discussion #4 Answers

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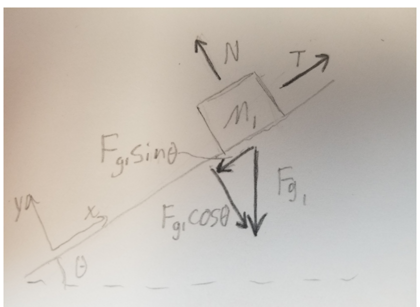
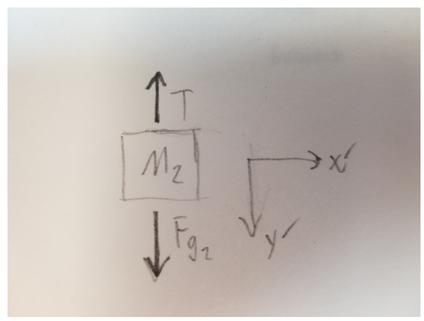
We should begin by first analyzing the forces acting on each block. The following is a force diagram for each of the two blocks:





Note that each block has it's own coordinate system. They were chosen for convenience, and when we apply Newton's laws we will see why these are convenient. For M_1 note that we broke up the force of gravity into x and y components. We need to do this before applying Newton's laws. Applying Newton's laws gives the following equations for each block:

M_1:

$$x: \quad T - F_{g1}\sin\theta = M_1 a_1 \tag{1}$$

$$y: \quad N - F_{g1}\cos\theta = 0 \tag{2}$$

M_2:

$$y': \quad F_{g2} - T = M_2 a_2 \tag{3}$$

Since the blocks are connected by the string that means that a_1 and a_2 have to be equal in magnitude. If M_2 falls down then M_1 moves up the ramp, and when relating the accelerations the signs should reflect this behavior. The way that I have defined my coordinate systems leads to the following relation:

$$a_1 = a_2 = a \tag{4}$$

The force of gravity is simply the acceleration of gravity multiplied by the mass of the object being acted upon. This gives us the following equations:

$$F_{g1} = gM_1 \tag{5}$$

$$F_{g2} = gM_2 \tag{6}$$

Plugging in (4), (5), and (6) into (1) and (3) gives the following two equations:

$$T - M_1 g \sin\theta = M_1 a \tag{7}$$

$$M_2 g - T = M_2 a \tag{8}$$

We can use these two equations to solve for a. Let's do this by solving for T in (7), plugging into (8), and solving the resulting equation. Solving for T with (7) gives:

$$T = M_1 a + M_1 g \sin\theta \tag{9}$$

Plugging (9) into (8) gives us an equation we can solve for the acceleration with:

$$\begin{aligned} &M_2 g - M_1 a - M_1 g \sin\theta = M_2 a \\ &\implies aM_1 + aM_2 = M_2 g - M_1 g \sin\theta \\ &\implies a = \frac{g(M_2 - M_1 \sin\theta)}{M_1 + M_2} \end{aligned} \tag{10}$$

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Now we want to find a relation between the masses such that the masses are at rest when released from rest. If the masses are at rest, then there is no acceleration. So to find this relation we can set (10) equal to zero and solve for one of the masses. Doing so gives the following relation.

$$\begin{aligned} 0 &= \frac{g(M_2 - M_1 \sin\theta)}{M_1 + M_2} \\ &\implies 0 = g(M_2 - M_1 \sin\theta) \\ &\implies M_2 = M_1 \sin\theta \end{aligned}$$