Physics 3210 Spring 2019 Discussion #3 Answers

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We could begin by finding the times where the bead is in the positions the question is asking about and then plug those times into the velocity and acceleration equations, but there is an easier way. If we substitute ωt with θ in the velocity and acceleration equations we get them in terms of the angle of rotation, which is easily seen from the graph given to us. Doing this substitution gives the following equations:

$$\overrightarrow{v} = u\hat{r} + u\theta\hat{\theta} \tag{1}$$

$$\overrightarrow{a} = -u\omega\theta\hat{r} + 2u\omega\hat{\theta} \tag{2}$$

Now we have to worry about converting $\hat{r}$ and $\hat{\theta}$ into $\hat{x}$ and $\hat{y}$ components. Here it is useful to see that all of the points of interest lie on the x or y axis. Considering this, there are only four conversions we will have to use:

1. For a point on the positive side of the x axis:

$$\hat{r} \longrightarrow \hat{x} \quad \& \quad \hat{\theta} \longrightarrow \hat{y}$$

2. For a point on the negative side of the x axis:

$$\hat{r} \longrightarrow -\hat{x} \quad \& \quad \hat{\theta} \longrightarrow -\hat{y}$$

3. For a point on the positive side of the y axis:

$$\hat{r} \longrightarrow \hat{y} \quad \& \quad \hat{\theta} \longrightarrow -\hat{x}$$

4. For a point on the negative side of the y axis:

$$\hat{r} \longrightarrow -\hat{y} \quad \& \quad \hat{\theta} \longrightarrow \hat{x}$$

Now we simply plug in appropriate values, angles, and conversions into (1) and (2) to find the x and y components of the bead's velocity and acceleration. The following is a table with the velocity and acceleration at each point of interest:

point	θ	velocity	acceleration
1	π	$u(-\hat{x}) + u\theta(-\hat{y}) = -\hat{x} - \pi\hat{y}$	$-u\omega\theta(-\hat{x}) + 2u\omega(-\hat{y}) = \pi^2\hat{x} - 2\pi\hat{y}$
2	$\frac{3\pi}{2}$	$u(-\hat{y}) + u\theta(\hat{x}) = \frac{3\pi}{2}\hat{x} - \hat{y}$	$-u\omega\theta(-\hat{y}) + 2u\omega(\hat{x}) = 2\pi\hat{x} + \frac{3\pi^2}{2}\hat{y}$
3	2π	$u(\hat{x}) + u\theta(\hat{y}) = \hat{x} + 2\pi\hat{y}$	$-u\omega\theta(\hat{x}) + 2u\omega(\hat{y}) = -2\pi^2\hat{x} + 2\pi\hat{y}$
4	$\frac{5\pi}{2}$	$u(\hat{y}) + u\theta(-\hat{x}) = -\frac{5\pi}{2}\hat{x} + \hat{y}$	$-u\omega\theta(\hat{y}) + 2u\omega(-\hat{x}) = -2\pi\hat{x} - \frac{5\pi^2}{2}\hat{y}$
5	3π	$u(-\hat{x}) + u\theta(-\hat{y}) = -\hat{x} - 3\pi\hat{y}$	$-u\omega\theta(-\hat{x}) + 2u\omega(-\hat{y}) = 3\pi^2\hat{x} - 2\pi\hat{y}$
6	$\frac{7\pi}{2}$	$u(-\hat{y}) + u\theta(\hat{x}) = \frac{7\pi}{2}\hat{x} - \hat{y}$	$-u\omega\theta(-\hat{y}) + 2u\omega(\hat{x}) = 2\pi\hat{x} + \frac{7\pi^2}{2}\hat{y}$

Now that we have the velocity and acceleration of the bead at the points of interest in Cartesian coordinates, we can graph them on the graph of the bead's trajectory. The following is the trajectory graph with scaled down velocity and acceleration vectors: