Physics 3210 Spring 2019 Discussion #2 Answers

Josh Peterson

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1.1

We can obtain the skier's x and y coordinates as functions of time using the following kinematic equation:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

For convenience let us define a Cartesian coordinate system with the x axis going right, the y axis going up, and the origin at the end of the ramp. If we use this coordinate system then $x_0 = y_0 = 0$. The initial velocity in the x and y direction is given by the x and y components of the initial velocity vector V. Using simple trigonometry we find these components to be:

$$V_x = V\cos(\theta)$$
$$V_y = V\sin(\theta)$$

There is no acceleration in the x direction and in the y direction we have $a = -g$ where g is the acceleration of gravity. So the kinematic equations are the following:

$$x(t) = V\cos(\theta)t \tag{1}$$

$$y(t) = V\sin(\theta)t - \frac{1}{2}gt^2 \tag{2}$$

We will now come up with an equation for the slope. The slope is a simple line with a y intercept of 0. The slope of the line is $\frac{y_2-y_1}{x_2-x_1}$ where (x_1, y_1) and (x_2, y_2) are two points on the line. With trigonometry we can show this to be $\pm\tan(\phi)$. The following is the line of the slope:

$$y(x) = -\tan(\phi)x \tag{3}$$

The skier will land when $y_{skier} = y_{slope}$. This is when the slope and skier functions intercept. Considering this, lets first solve for time as a function of x using (1):

$$\begin{aligned} x(t) &= V\cos(\theta)t \\ \iff t &= \frac{x}{V\cos(\theta)} \end{aligned} \tag{4}$$

Plugging (4) into equation (2) gives us the y position of the skier as a function of x, the same type of function as the slope function (3):

$$\begin{aligned} y(t) &= y\left(\frac{x}{V\cos(\theta)}\right) \\ &= V\sin(\theta)\frac{x}{V\cos(\theta)} - \frac{1}{2}g\left(\frac{x}{V\cos(\theta)}\right)^2 \\ &= \tan(\theta)x - \frac{gx^2}{2V^2\cos^2(\theta)} \end{aligned} \tag{5}$$

Now we can set functions (5) and (3) for the skier and the slope equal to each other to obtain the x position of the skier when the skier lands:

$$\begin{aligned} &-\tan(\phi)x_{landing} = \tan(\theta)x_{landing} - \frac{gx_{landing}^2}{2V^2\cos^2(\theta)} \\ &\iff \frac{-g}{2V^2\cos^2(\theta)}x_{landing}^2 + (\tan(\theta)+\tan(\phi))x_{landing} = 0 \\ &\iff x_{landing} = 0 \quad \text{or} \quad \frac{-g}{2V^2\cos^2(\theta)}x_{landing} + (\tan(\theta)+\tan(\phi)) = 0 \\ &\iff x_{landing} = \frac{2(\tan(\theta)+tan(\phi))V^2\cos^2(\theta)}{g} \\ &= \frac{2(\tan(20.0)+\tan(42.0))(12.0\frac{m}{s})^2\cos^2(20.0)}{9.8\frac{m}{s^2}} \\ &\approx 32.811m \end{aligned}$$

Plugging in this x value into equation (5) gives the following y position of when the skier lands:

$$\begin{aligned} y_{landing} &\approx \tan(20.0) * 32.811m - \frac{9.8\frac{m}{s^2} * (32.822m)^2}{2 * (12.0\frac{m}{s})^2 * \cos^2(20.0)} \\ &= -29.544m \end{aligned}$$

The distance travelled is then simply the following:

$$\begin{aligned} distance &= \sqrt{x_{landing}^2 + y_{landing}^2} \\ &\approx 44.152m \end{aligned}$$

1.2

To find the x and y components of velocity we can take a time derivative of equations (2) and (3):

$$v_x(t) = \frac{dx}{dt}(t) = V\cos(\theta) \tag{6}$$

$$v_y(t) = \frac{dy}{dt}(t) = V\sin(\theta) - gt \tag{7}$$

We see immediately that the x component of velocity is constant so plugging V and θ into (6) will give us the x component of the landing velocity. To find the y component of the landing velocity all we have to do is find the time of landing and plug that time into (7). We can easily obtain the time of landing by taking $x_{landing}$ and plugging that into (4):

$$t_{landing} = \frac{x_{landing}}{V\cos(\theta)}$$

Plugging this into (6) gives:

$$v_y(t_{landing}) = V\sin(\theta) - g\frac{x_{landing}}{V\cos(\theta)} \tag{8}$$

Plugging appropriate values into (6) and (8) gives us the horizontal and vertical components of the velocity right before landing:

$$\begin{aligned} v_y &\approx (12\frac{m}{s})\sin(20.0) - (9.8\frac{m}{s^2}\frac{32.811m}{(12\frac{m}{s})\cos(20.0)} \\ &\approx -24.411\frac{m}{s} \end{aligned}$$

$$\begin{aligned} v_x &= (12.0\frac{m}{s})\cos(20.0) \\ &\approx 11.276\frac{m}{s} \end{aligned}$$